UC Berkeley
Mechanical Engineering Department

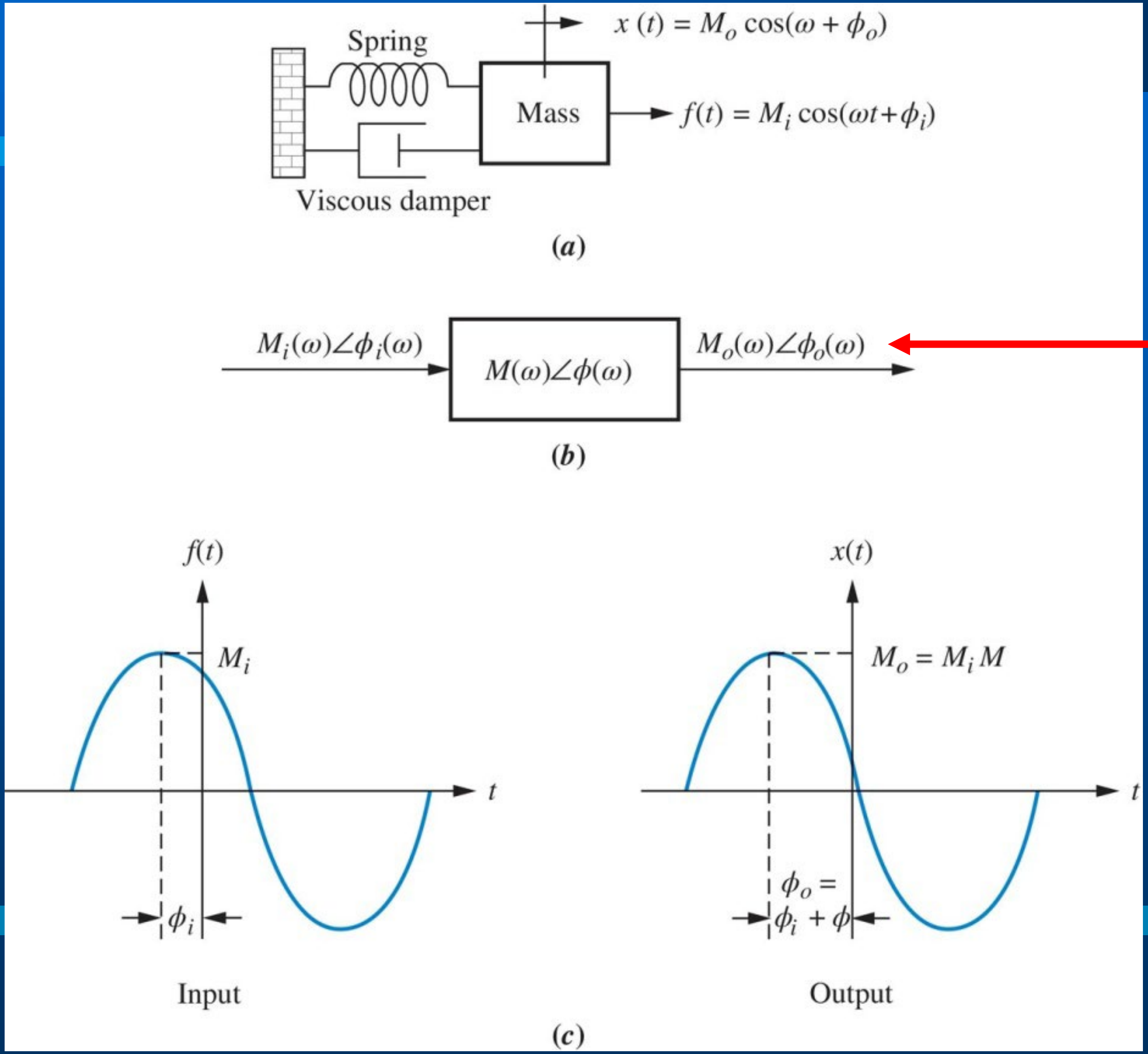
ME103:: Experimentation and Measurements

Lecture #9

Concept of Frequency Response:

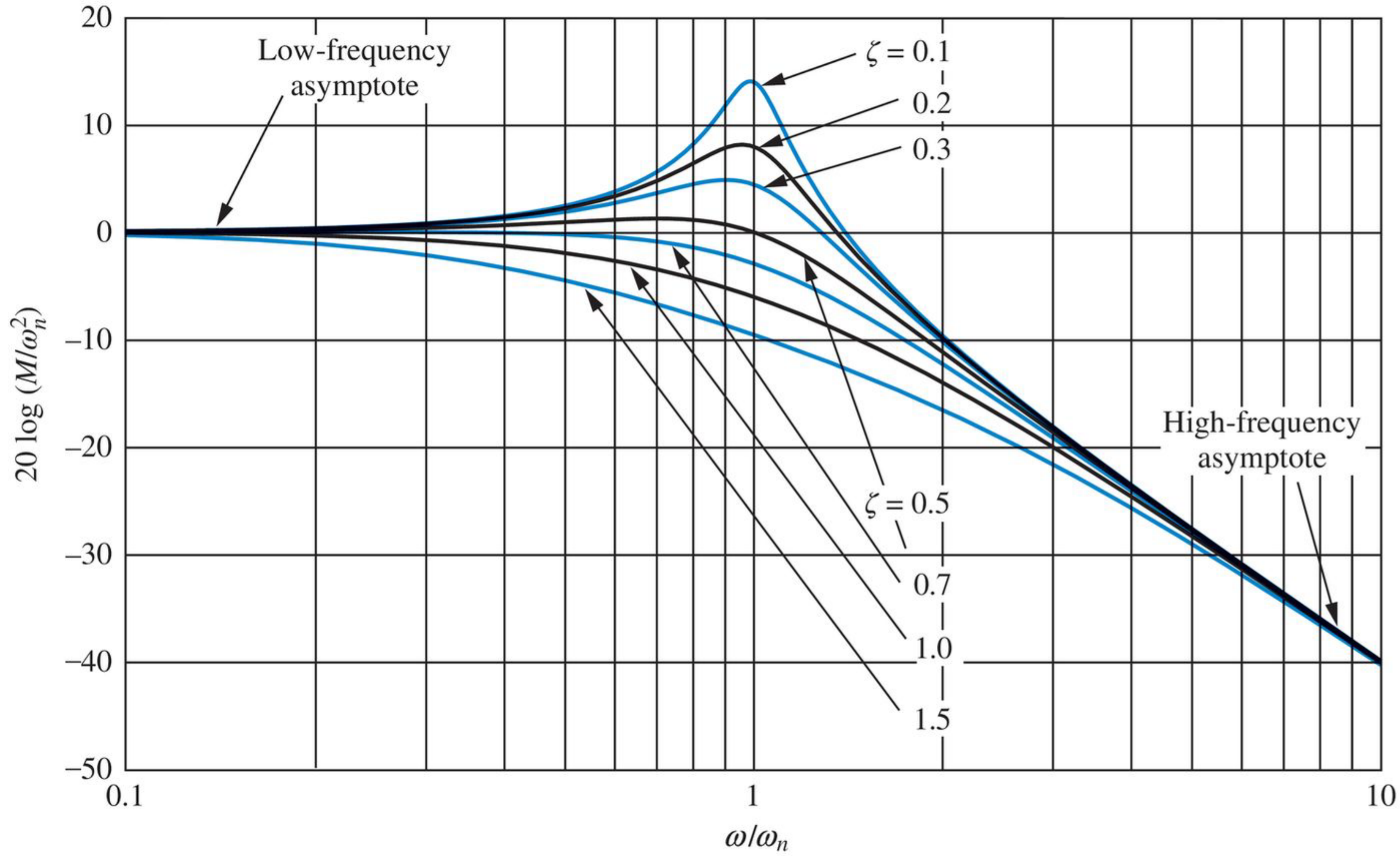
- *If the system is Linear Time Invariant (LTI)*
  - *In Steady State, sinusoidal input will generate a sinusoidal response of the same frequency*
  - *Although the frequency is the same, the response differ in magnitude and phase*

Frequency Response and the Bode Plot:

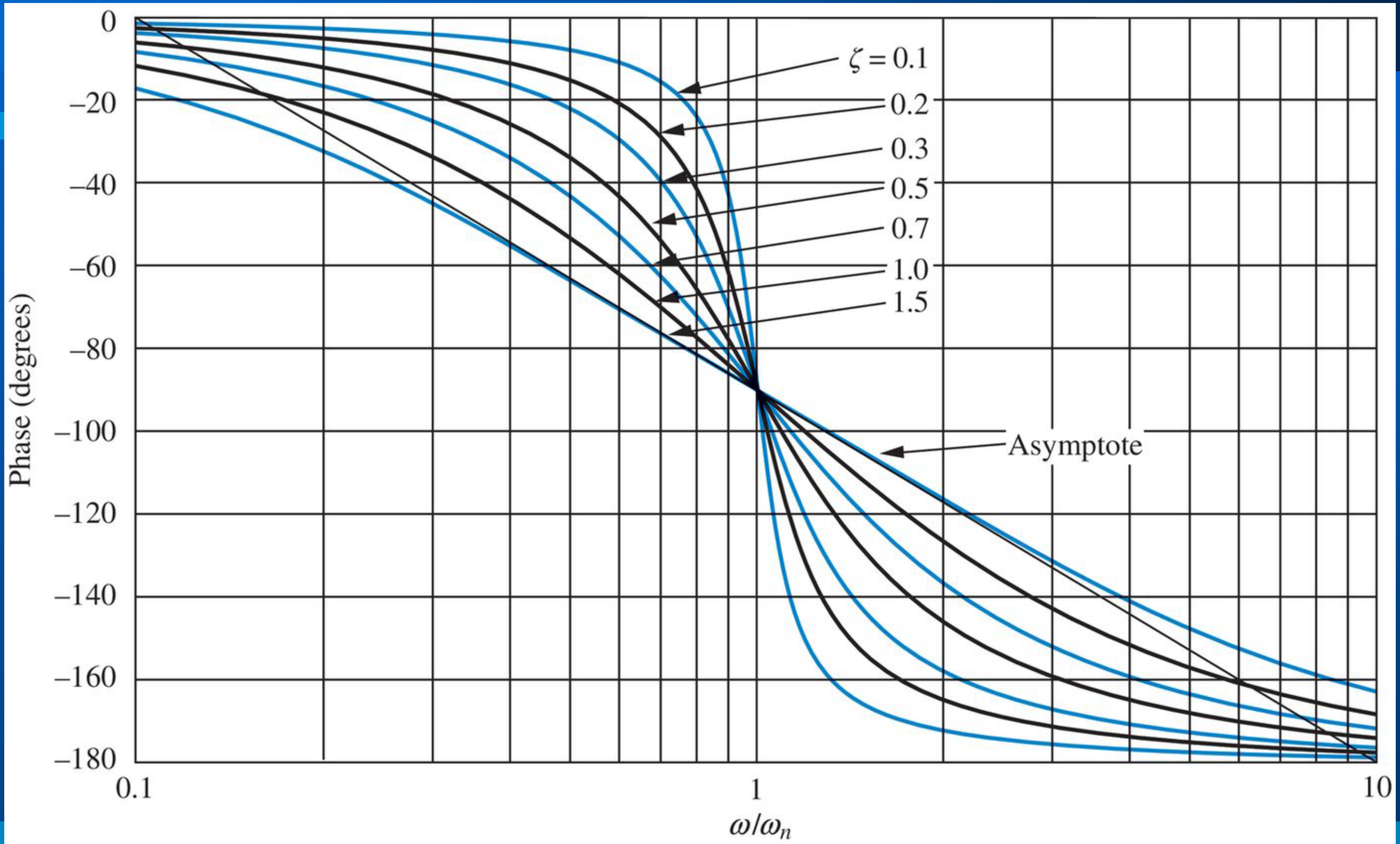


Phasor Notation

Bode Plot:


20
10
0
−10
−20
−30
−40
−50
0.1
1
10
20 log (M/ω_n^2)
ω/ω_n
Low-frequency asymptote
High-frequency asymptote
ζ = 0.1
0.2
0.3
ζ = 0.5
0.7
1.0
1.5

Bode Plot:


0
–20
–40
–60
–80
–100
–120
–140
–160
–180
Phase (degrees)
0.1
1
10
ω/ω_n
ζ = 0.1
0.2
0.3
0.5
0.7
1.0
1.5
Asymptote

Bode Plot:

$$\frac{dx(t)}{dt} = u(t)$$

$$u(t) = \cos(\omega t)$$

$$x(t) = \frac{1}{\omega}\sin(\omega t) = \frac{1}{\omega}\cos(\omega t - \frac{\pi}{2})$$

$$M(\frac{x(t)}{u(t)}) = \left|\frac{1}{\omega}\right|$$

AC Circuit Analysis using Phasors and Impedances

$$V_s(t) = V_A \cos(\omega(t) + \varphi)$$

Phasor Notations:

(1) $\overline{V_s} = V_A \angle \varphi$ standard form

(3) $\overline{V_s} = V_A e^{j\varphi}$ exponential form

(2) $\overline{V_s} = a + jb$ complex form

$$a = V_A \cos \varphi$$
$$b = V_A \sin \varphi$$

Converting complex to standard:

$$V_A = \sqrt{a^2 + b^2}$$

$$\varphi = \tan^{-1}(\frac{b}{a})$$

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Example:

Express $V_s(t) = 3\sin(100(t) + \ 40^o)$ in standard and complex phasor form

$$V_s(t) = 3\cos(100(t) + \ 40^o \ - 90^o)$$
$$V_s(t) = 3\angle -50^o$$ standard form

$$a = 3\cos(-50^o) = 1.928$$
$$b = 3\sin(-50^o) = 2.298$$

$$V_s(t) = 1.928 - j2.298$$ complex form

Bode Plot:

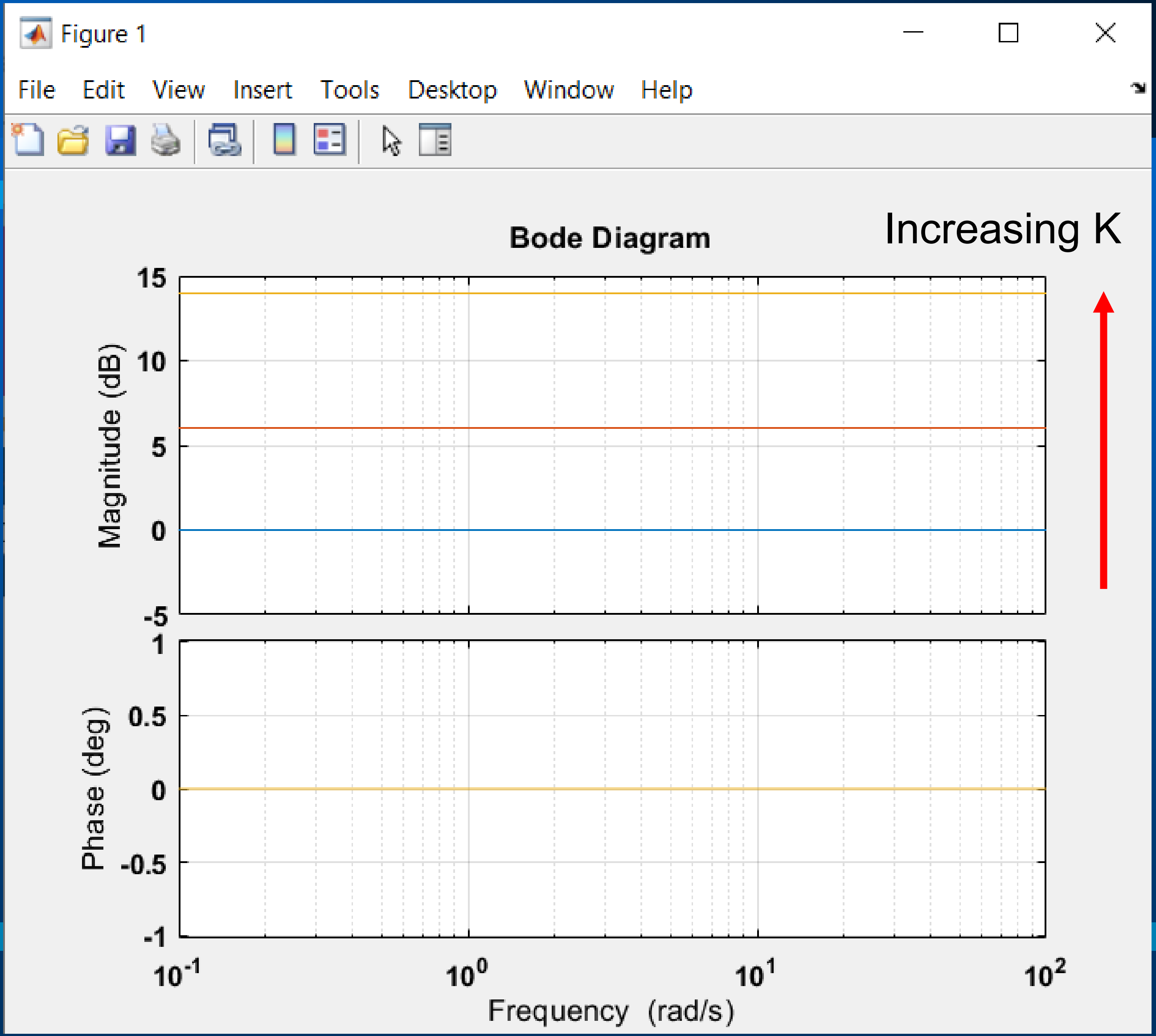
MATLAB R2020b - academic use

HOME PLOTS APPS

C: ▸ Users ▸ ms

Command Window

```
>> sys1 = tf([1],[1])

sys1 =

  1

Static gain.

>> bode(sys1)
>> hold
Current plot held
>> bode(2*sys1)
>> bode(5*sys1))
```



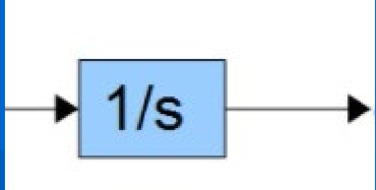
$|K|$

$$20log|1| = 0$$

$$20log|2| \approx 6.02$$

$$20log|5| \approx 13.98$$

Bode Plot:

$u(t)$  $x(t)$

$$x(t) = \int_{0^-}^{t} u(t)dt$$

$$\frac{dx(t)}{dt} = u(t)$$

$$u(t) = \cos(\omega t)$$

$$x(t) = \frac{1}{\omega}\sin(\omega t) = \frac{1}{\omega}\cos(\omega t - \frac{\pi}{2})$$

Bode Plot:

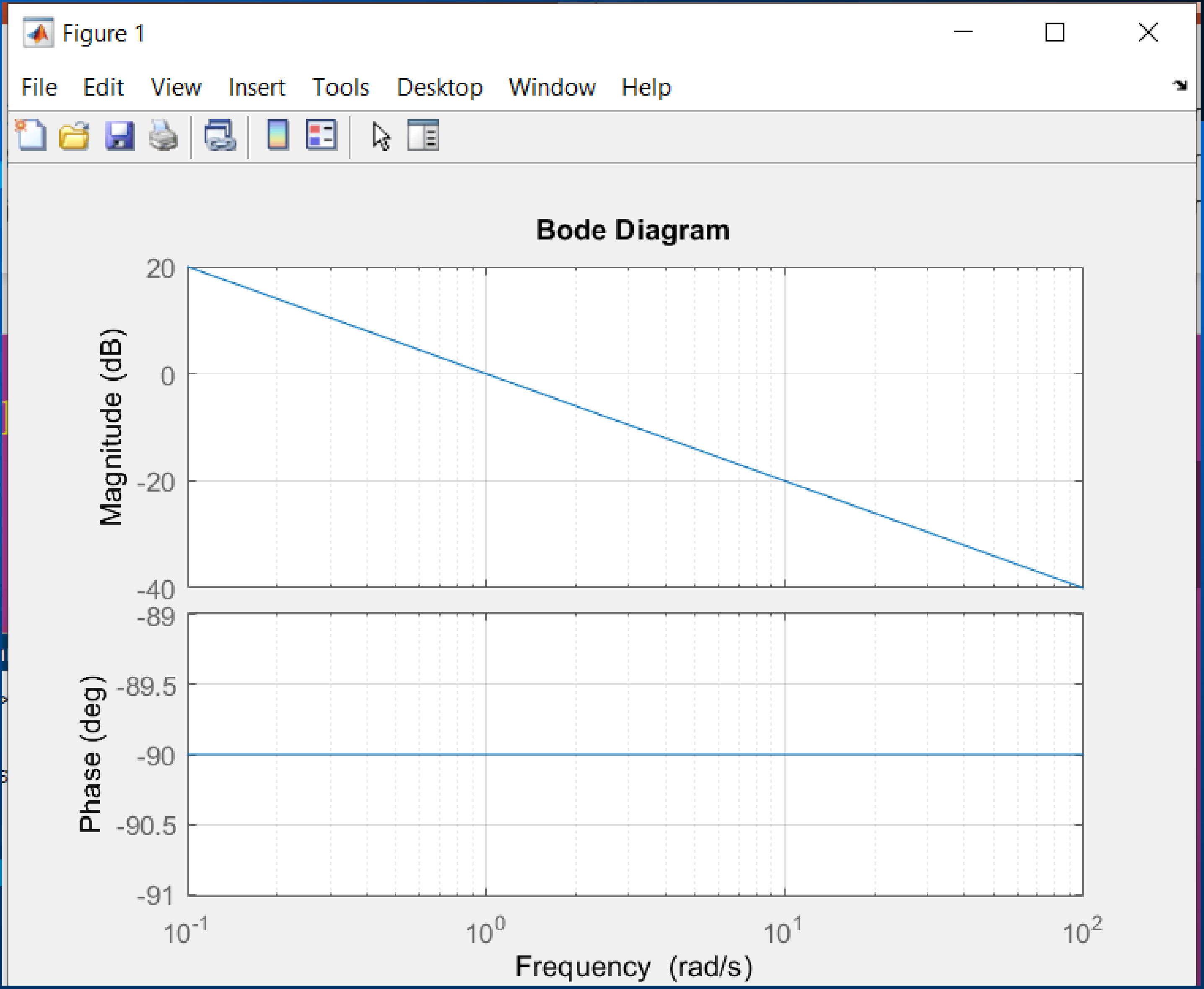
Command Window

```
>> sys1 = tf([1],[1 0])

sys1 =

  1
  -
  s

Continuous-time transfer function.

>> bode(sys1)
>>
```

$$j\omega \rightarrow s$$

$$\frac{O(j\omega)}{I(j\omega)} = \left|\frac{1}{j\omega}\right|$$

$$M(j\omega) = 0 - \frac{1}{\omega} j$$



AC Circuit Analysis using Phasors and Impedances

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$$a = V_A \cos\varphi$$
$$b = V_A \sin\varphi$$

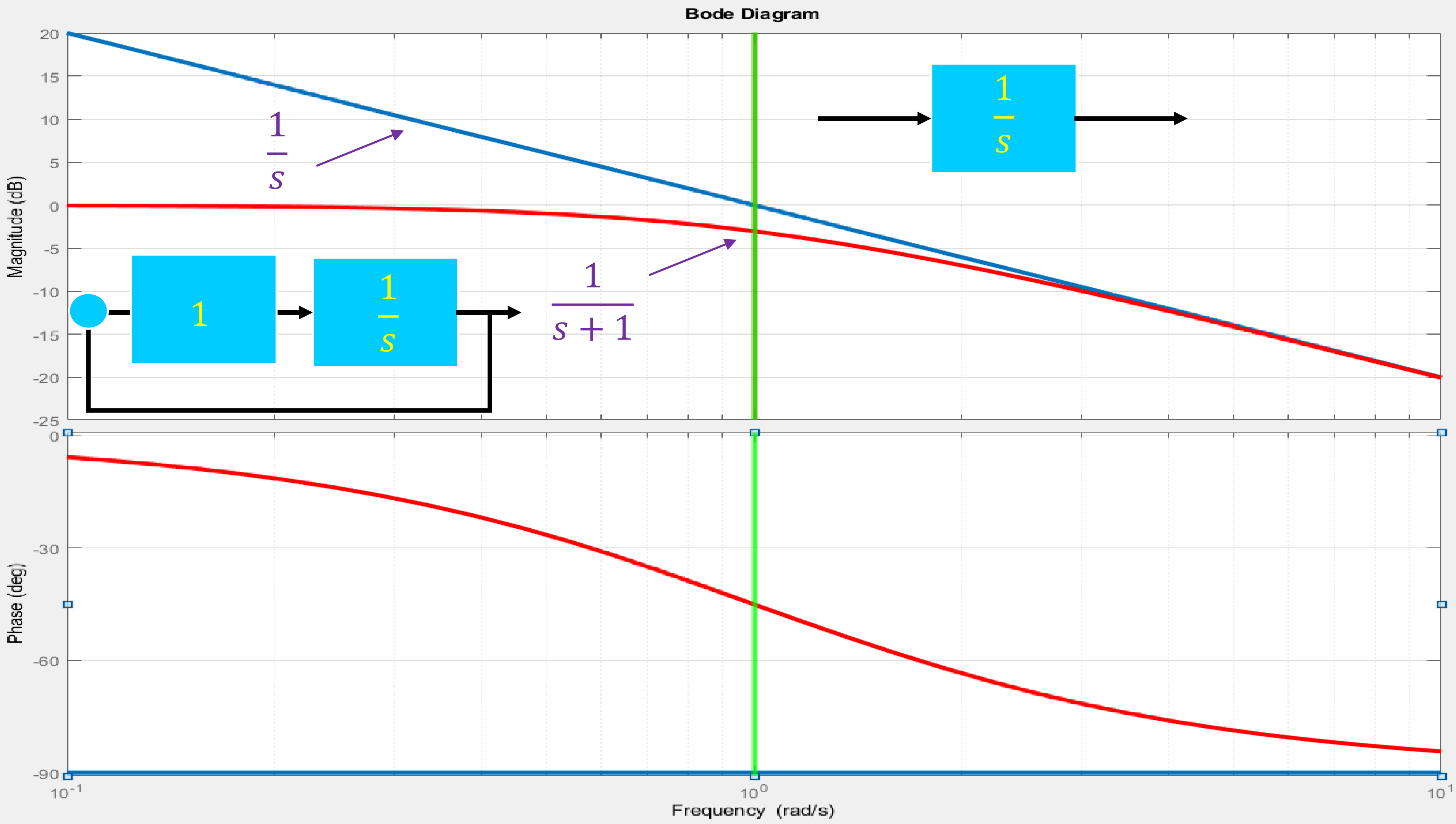
(3) $\overline{V_s} = V_A e^{j\varphi}$ exponential form

Converting complex to standard:

$$V_A = \sqrt{a^2 + b^2} = \sqrt{0^2 + (-\frac{1}{\omega}j)^2 = j\left|\frac{1}{\omega}\right|}$$

$$\varphi = \tan^{-1}(\frac{b}{a}) = \tan^{-1}\frac{-\frac{1}{\omega}}{0} = -\frac{\pi}{2}$$

Using Bode Plot to design (Frequency Shaping):

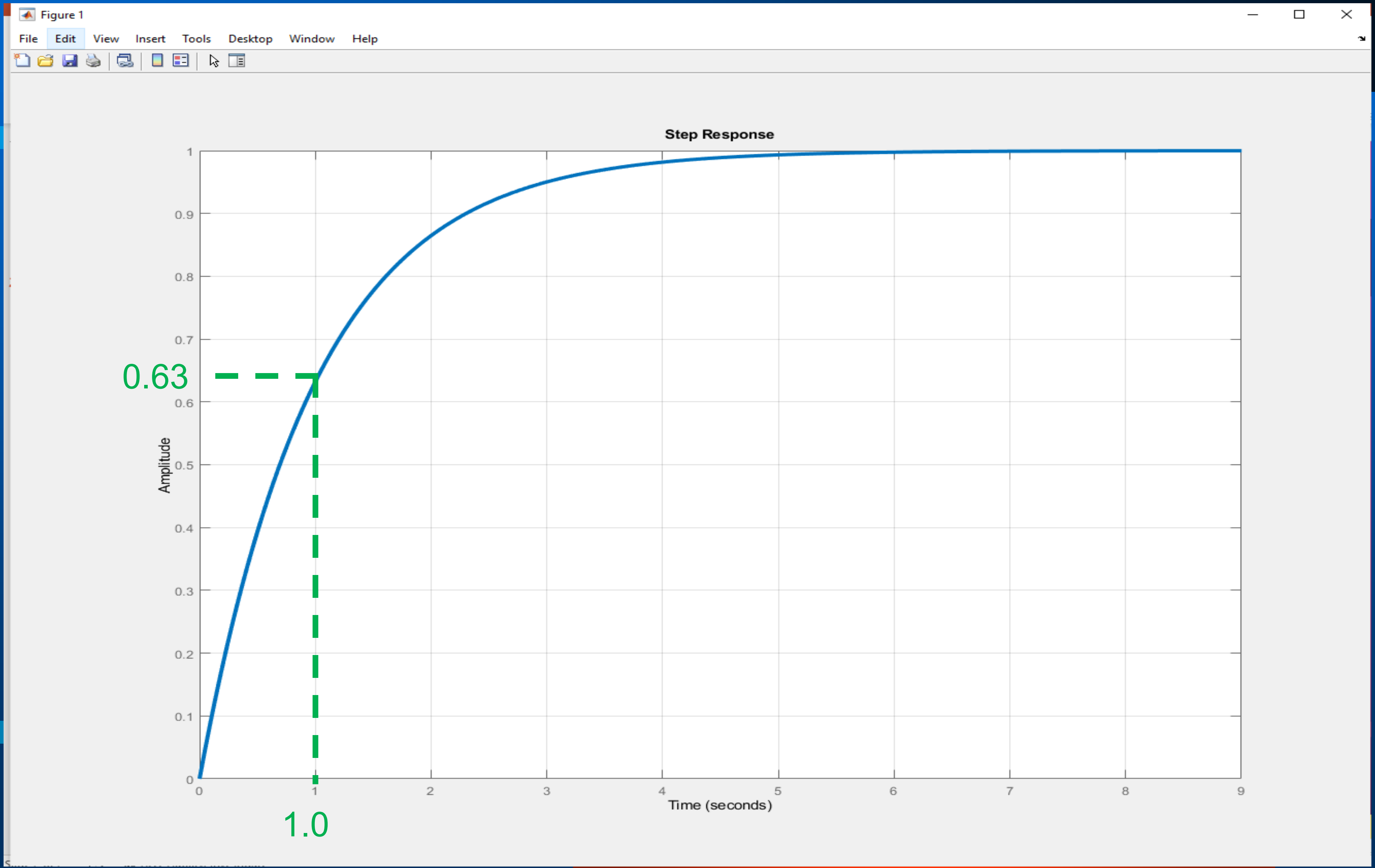


Using Bode Plot to design (Frequency Shaping):

Closed Loop TF:

$$\frac{1}{s+1}$$

$$c(t) = 1 - e^{-t}$$

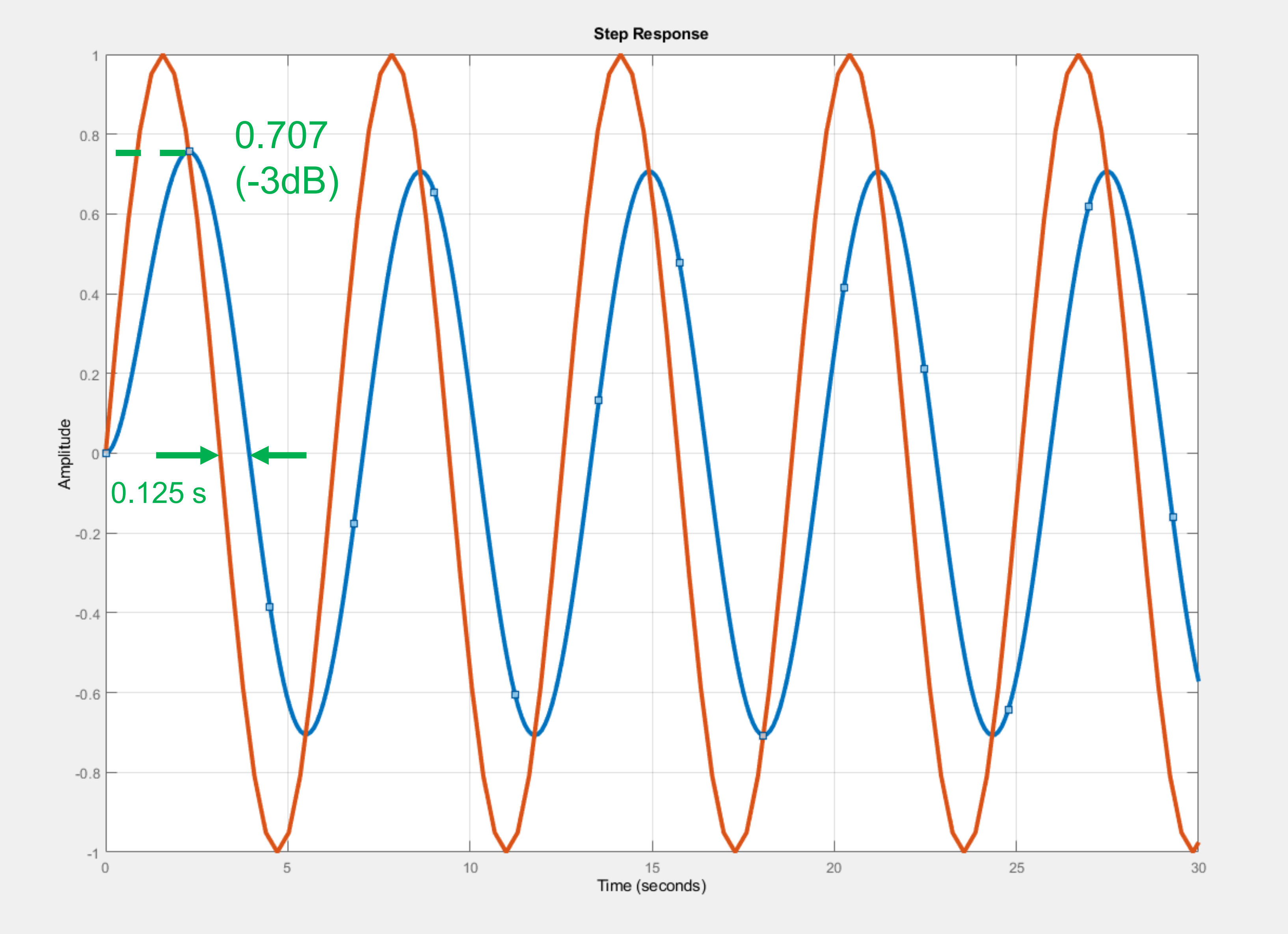


Using Bode Plot to design (Frequency Shaping):

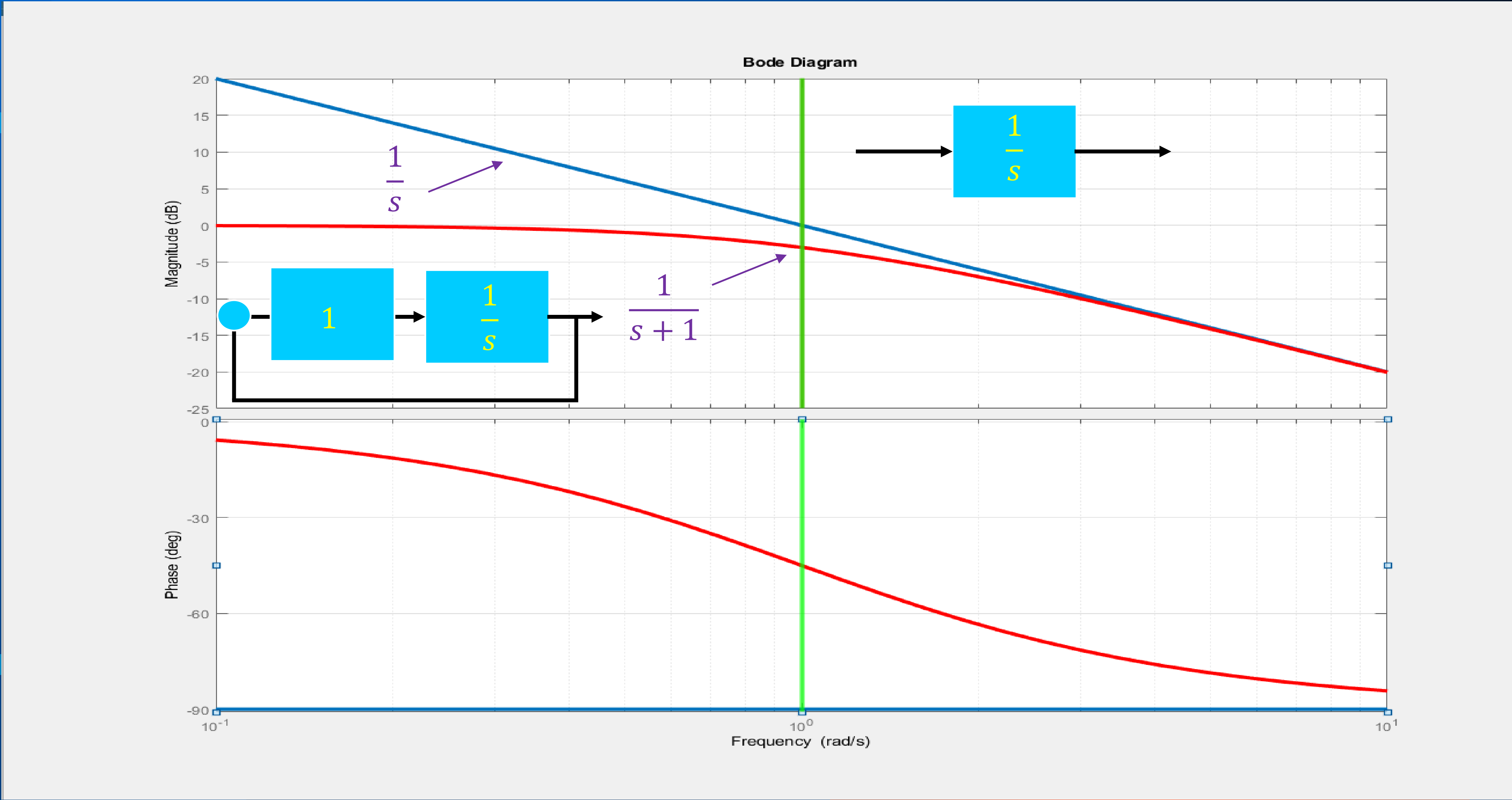
$@\omega = 1\, rad/s$

Open loop gain = 1

Closed loop gain = -3dB
Closed loop phase = -45º



Using Bode Plot to design (Frequency Shaping):

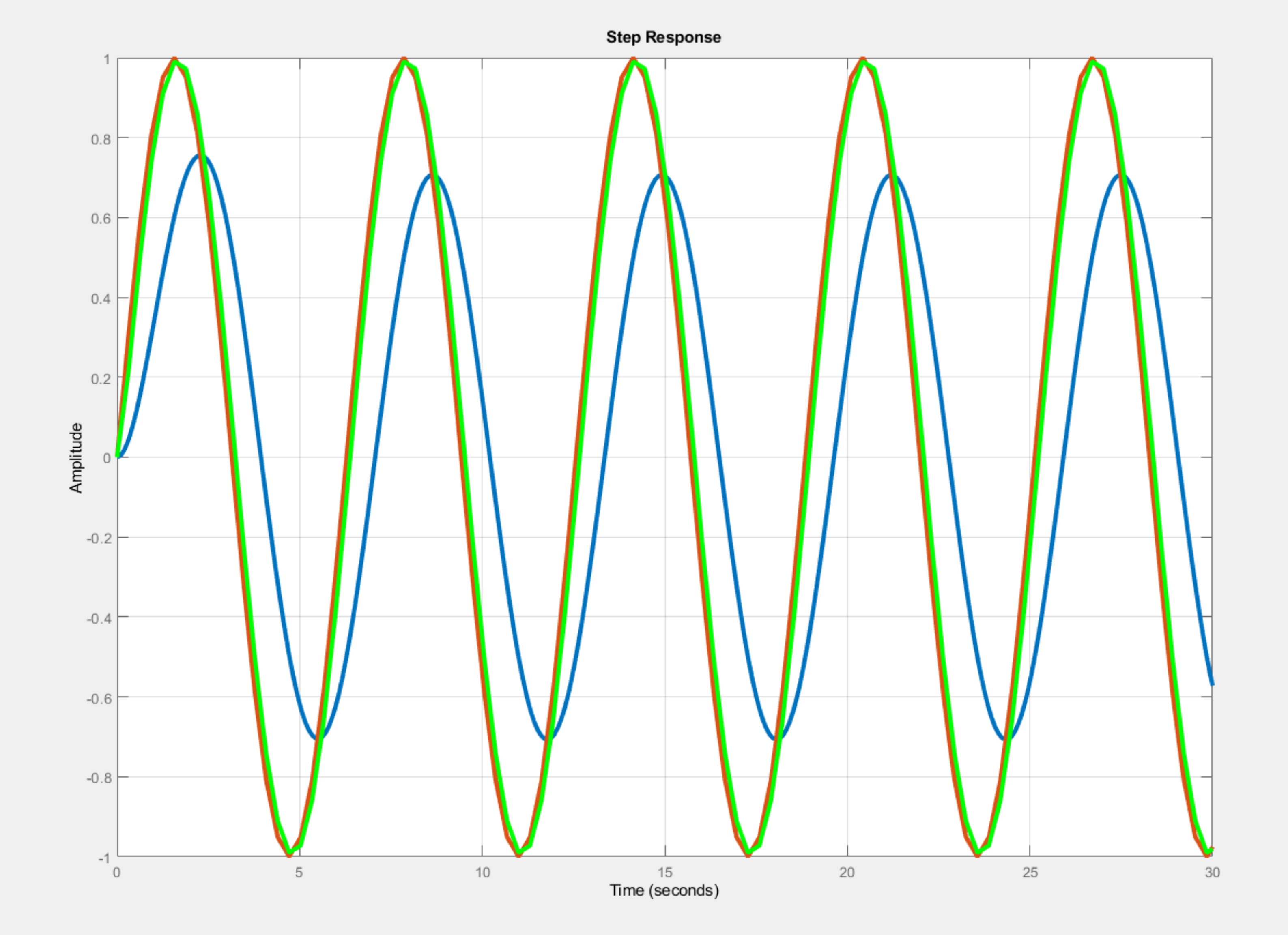


Using Bode Plot to design (Frequency Shaping):

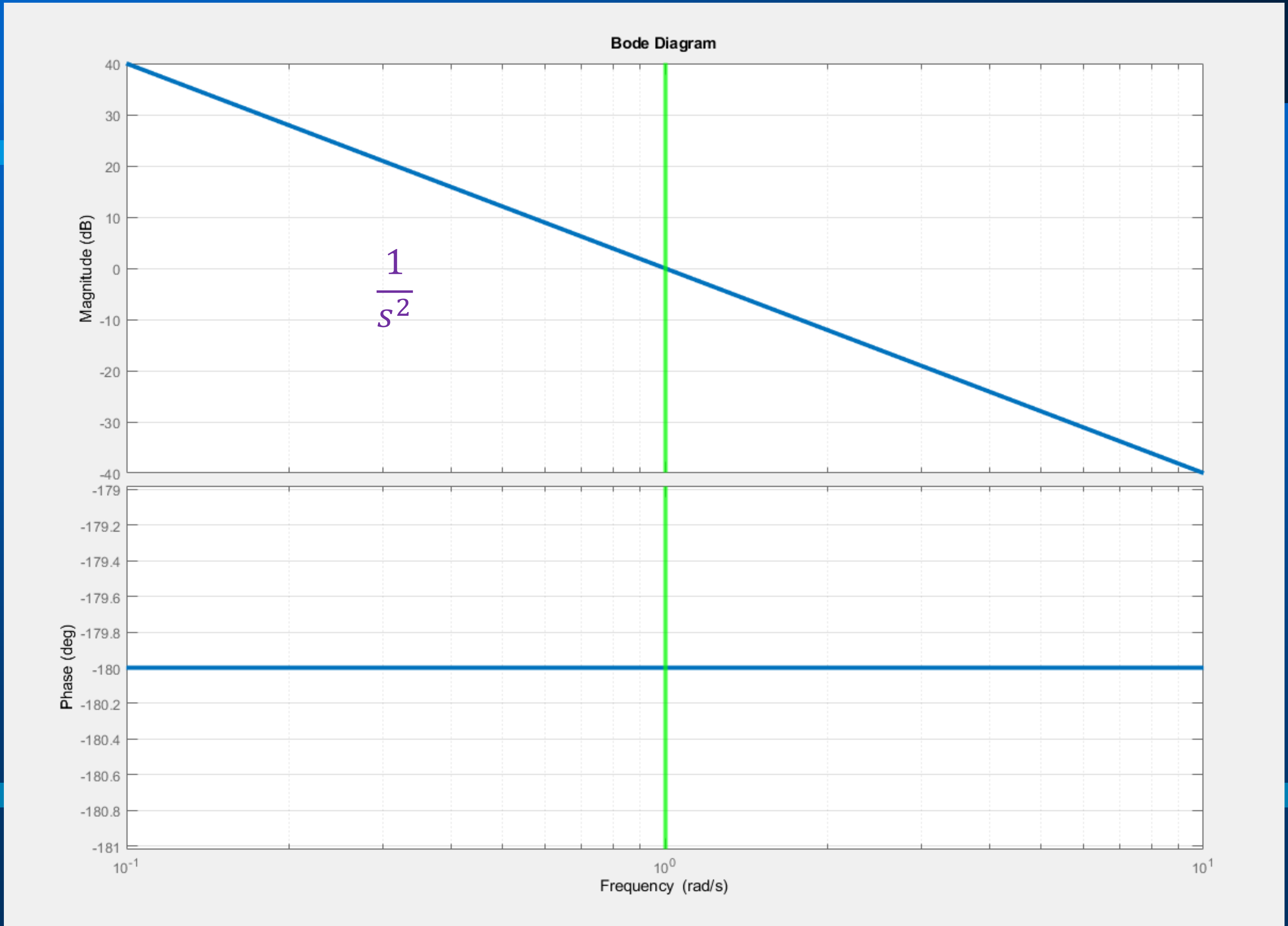
$@\omega = 1\, rad/s$

Open loop gain = 10 = 1dB

Closed loop gain = -3dB
Closed loop phase = 45º



Using Bode Plot to design (Frequency Shaping):



Using Bode Plot to design (Frequency Shaping):

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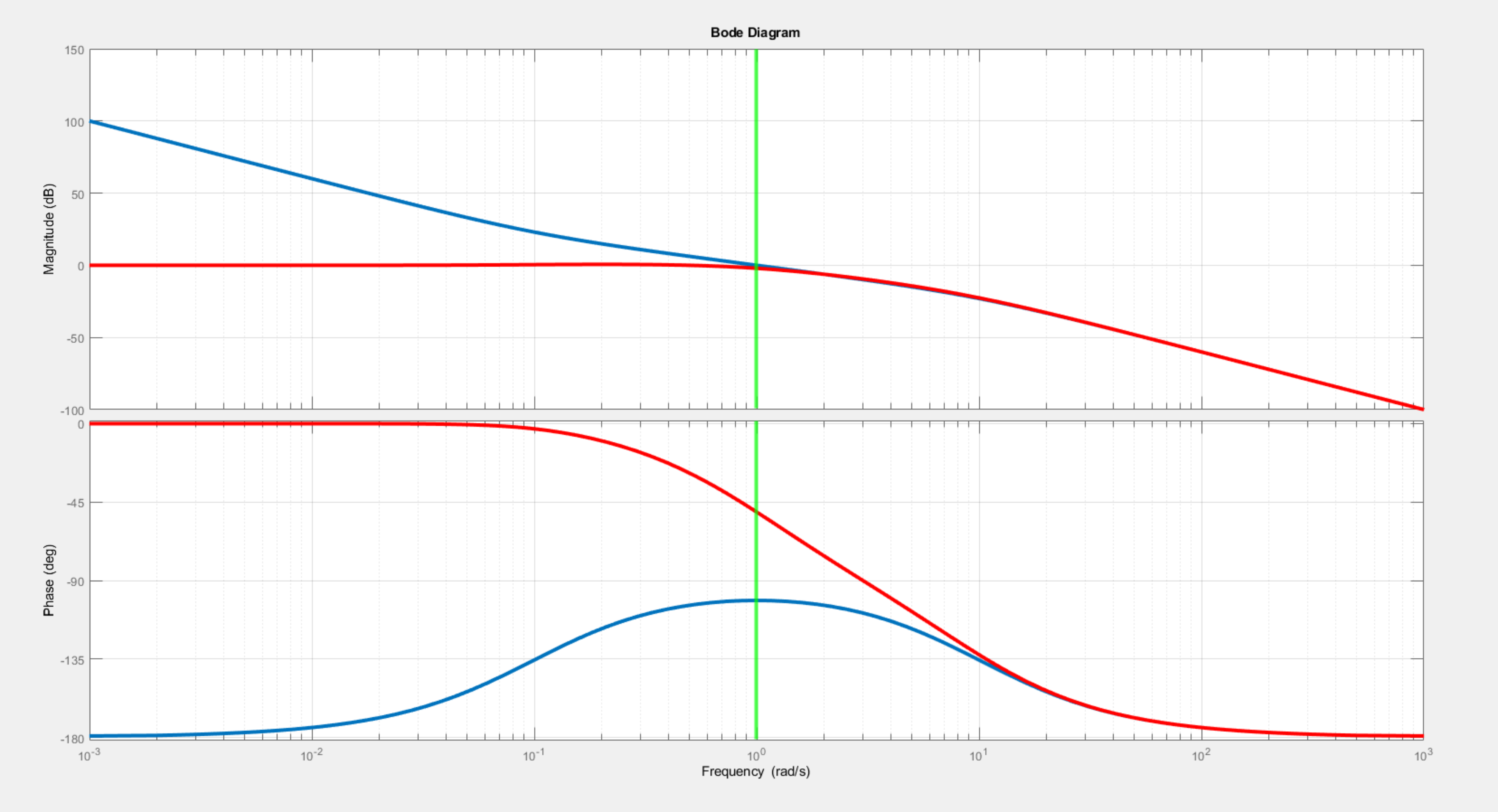
$$\frac{10(s+0.1)}{(s+10)}$$



Using Bode Plot to design (Frequency Shaping):