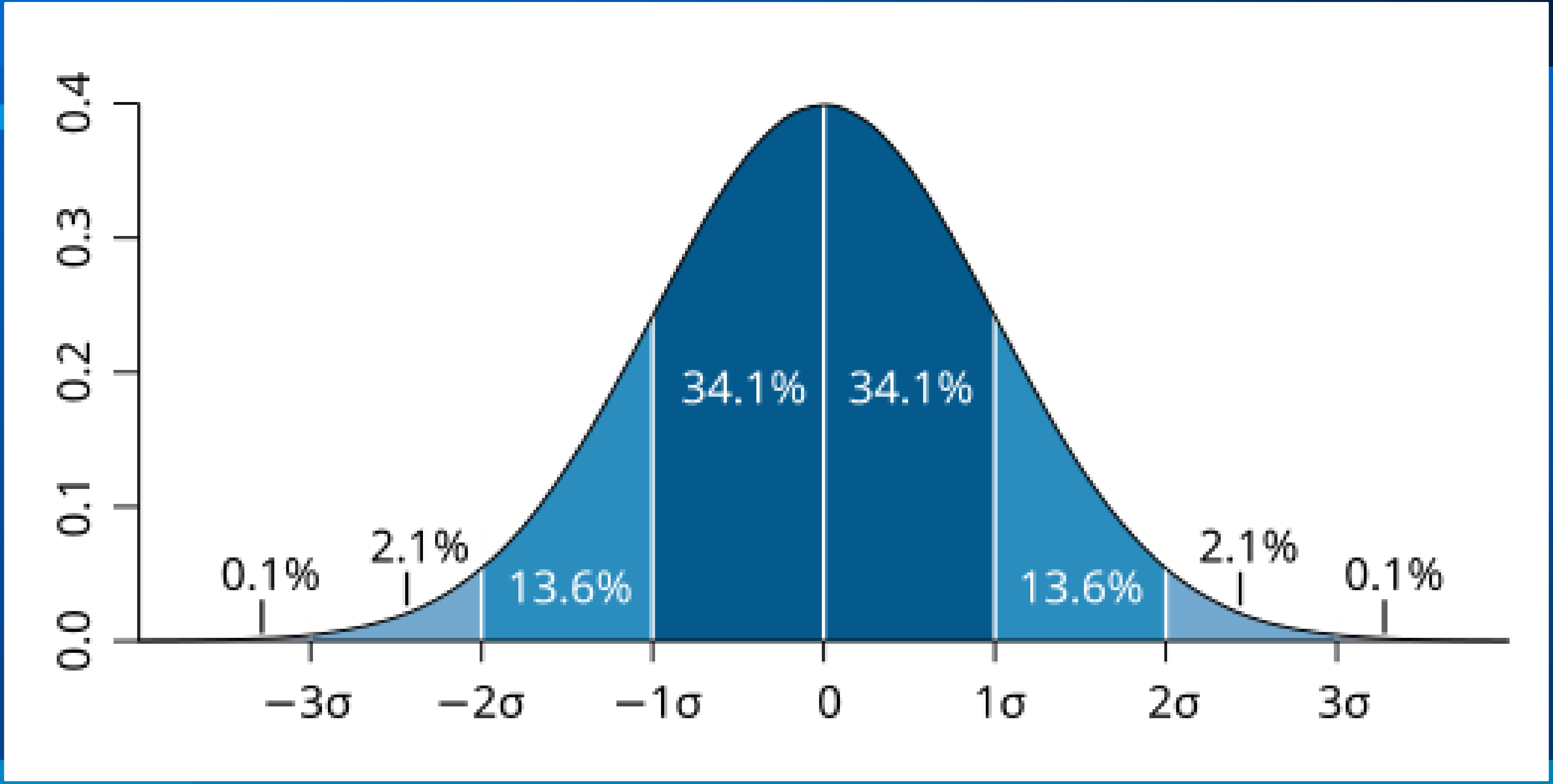
UC Berkeley
Mechanical Engineering Department

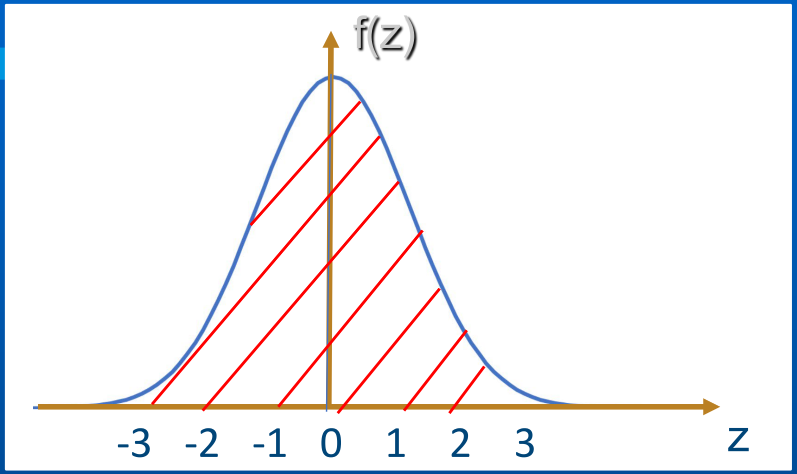
ME103:: Experimentation and Measurements

Lecture #5


0.4
0.3
0.2
0.1
0.0
0.1%
2.1%
13.6%
34.1%
34.1%
13.6%
2.1%
0.1%
−3σ
−2σ
−1σ
0
1σ
2σ
3σ

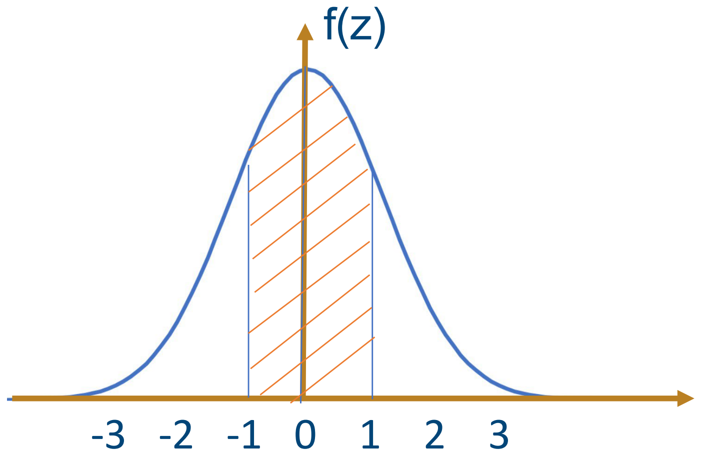
A characteristic of all Gaussians is that the total area under its curve is 1, i.e.

$$\int_{-\infty}^{+\infty} f(z)dz = 1$$



In fact, the area under the curve gives us the probability of occurrence or confidence level

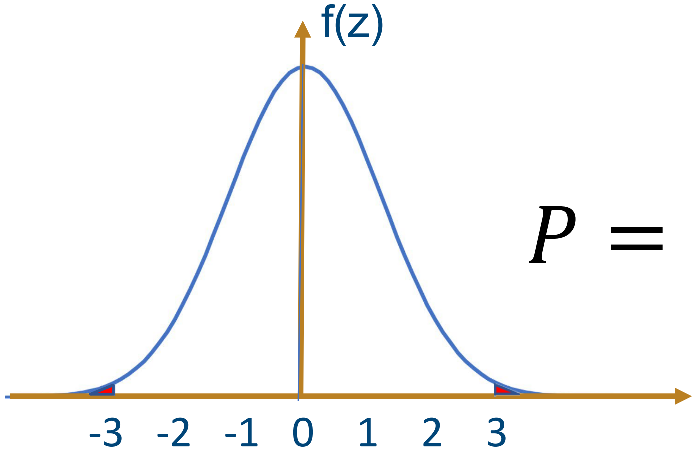
What if we were to limit ourselves to ±α ?



$$\int_{-1}^{+1} f(z)dz = 0.683$$

- Probability that your data lies between ±σ of the true value μ is 68.3%
- You are 68.3% confident that your data lies within ±σ of the true value

What is the probability that your data lies outside of ±3α?



$$P = 1 - \int_{-3}^{+3} f(z)dz = 0.003$$

z = |(x - μ)/σ|

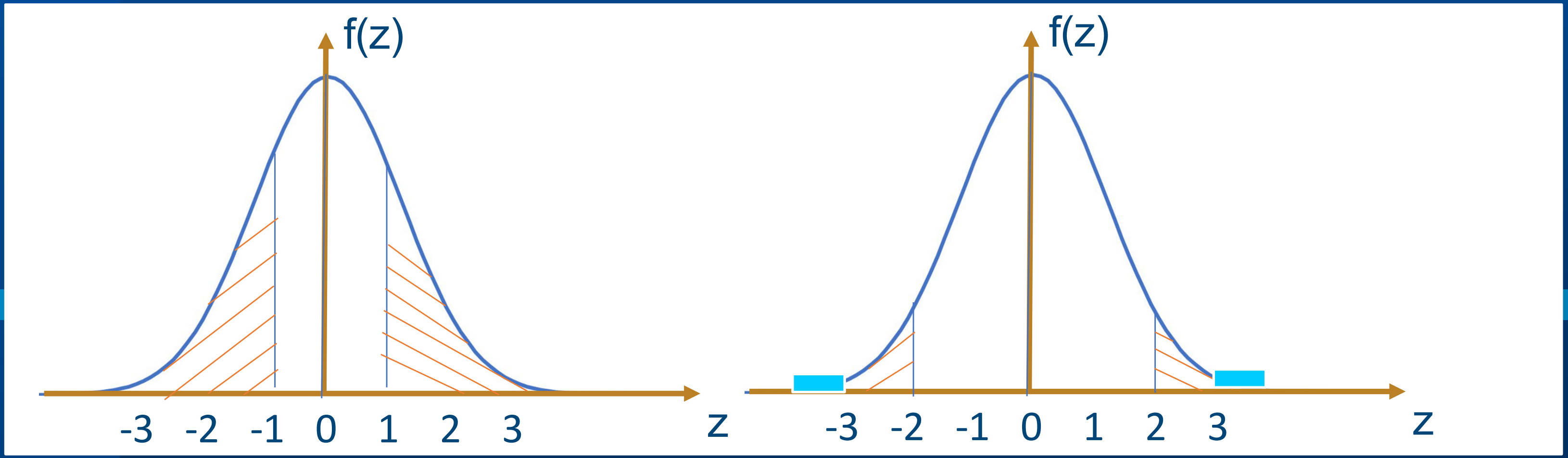
Probability that your data lies beyond ±3σ is 0.3%

Highly unlikely!

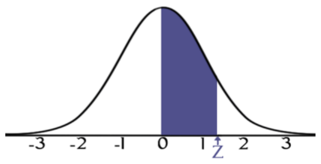
Outlier!

As an engineer, you need to make a decision on when to throw out your data:

Beyond $\pm\sigma$	➔	1 - 0.683 = 0.317 or 31.7%
$\pm 2\sigma$	➔	1 - 0.95 = 0.05 or 5%
$\pm 3\sigma$	➔	1 – 0.997 = 0.003 or 0.3%



Z-Distribution Table: Area under the Gaussian Curve…



STANDARD NORMAL TABLE (Z)

Entries in the table give the area under the curve between the mean and *z* standard deviations above the mean. For example, for *z* = 1.25 the area under the curve between the mean (0) and *z* is 0.3944.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0190	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2969	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3513	0.3554	0.3577	0.3529	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
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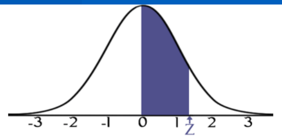
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998

$$z = \left| \frac{x - \mu}{\sigma} \right|$$

How do we use this table?

Z-Distribution Table *Area under the Gaussian Curve…*

STANDARD NORMAL TABLE (Z)



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Example

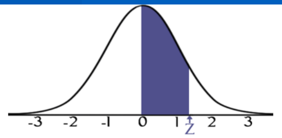
1) What is the area under the curve between z = -1.43 and z = 1.43?

   Total area = 2(0.4236) = 0.8472

2) What is the significance of this area?

   84.72% of the population lies within ± z (or ±1.43 sigma)

Z-Distribution Table *Area under the Gaussian Curve…*



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0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
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0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3513	0.3554	0.3577	0.3529	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
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2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998

Example #2

What range of x will contain 90% of the data?

Total area = 0.90

Table shows on ½ the area ➔ 0.45

z~1.645

$x = \mu \pm z\sigma$ ➔ $\mu - 1.645\sigma < x < \mu + 1.645\sigma$

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Error Analysis

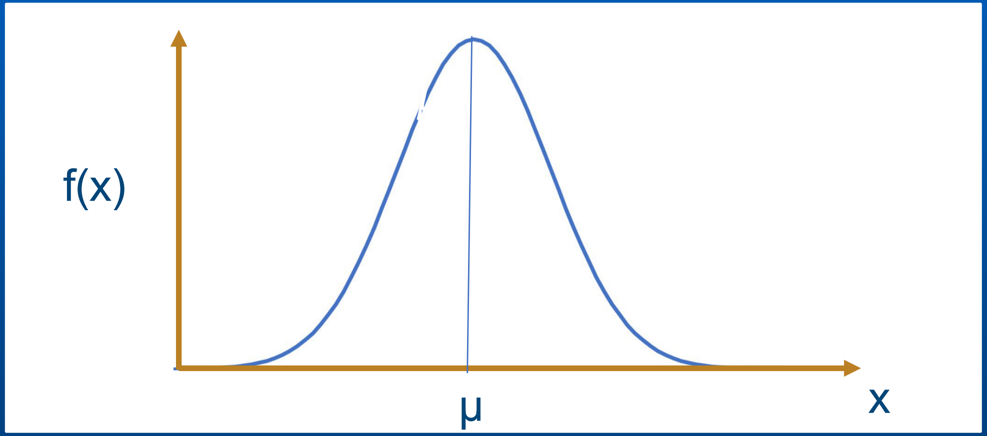
Everything that we talked up to now involved a large population, and we can accept that the mean value $\bar{x} \approx \mu$

But!!

How large is large???

1) Population Distribution

What we have been discussing all this time➔lots of data and binned into a histogram

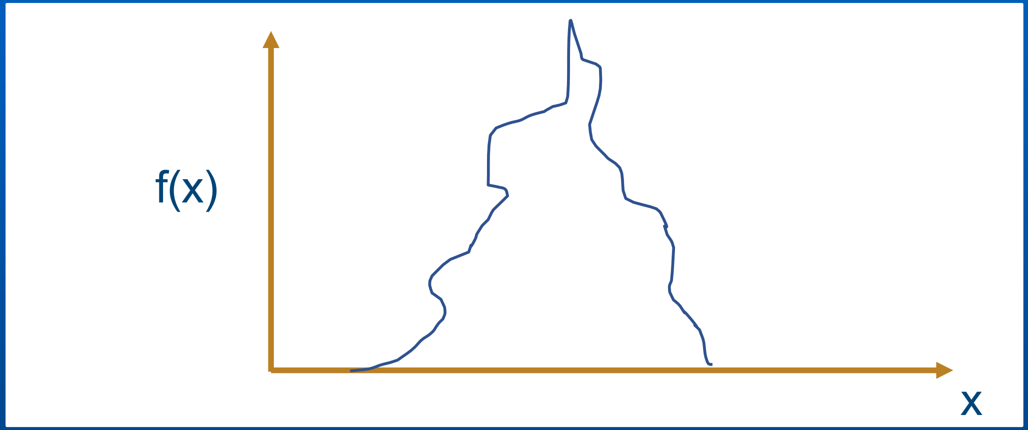


μ = true value (or if you have a **really large** population, $\mu \approx \bar{x}$)

0 = standard of deviation

2) Sample Distribution

Here, you select a sample of a population



$\bar{x}$ = average of sample mean

$$S_x = \sqrt{\frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \cdots + (x_n - \bar{x})^2}{n-1}}$$

S_x = standard deviation of sample

Note: (n-1) = degree of freedom

Two questions about the sample distribution…

Would you expect the sample mean $\mu = \overline{x}$?

Would you expect $S_x = \sigma$?

No!!!

But you would expect the values to be close, i.e.

$$\mu \approx \overline{x} \qquad S_x \approx \sigma$$

A series of questions…

- ✓ What happens if we change the sample size from which we construct the sampling distribution

➔The larger the sampling size, the better estimate of the population mean

- ✓ How are $\mu_{\bar{x}}$ and $\sigma_{\bar{x}}$ related to μ and σ, respectively?

➔$\mu_{\bar{x}} = \mu$ and $\sigma_{\bar{x}} = \sigma/\sqrt{n}$ where n is the sample size *not* the population

- ✓ Can your sampling size, n, be just a single element or number?

➔No. you need to take a sufficient sample of the population

Let's go back to our original question:

- **How large is large?**

Statistically, $n > 30$

Let’s go back to our original question:

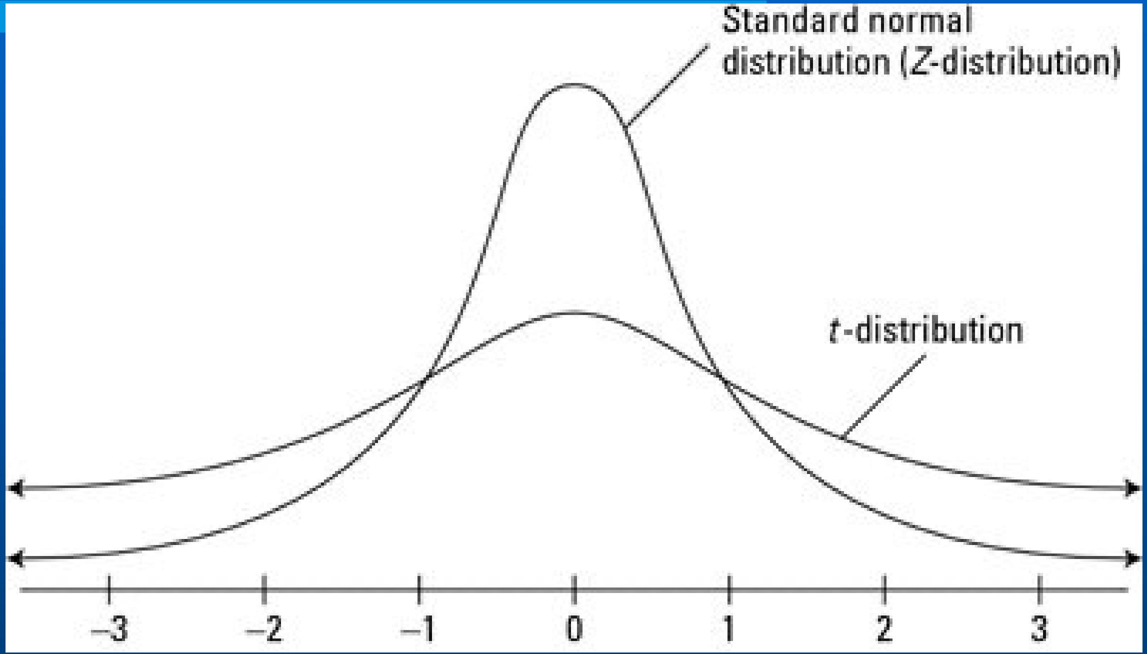
- **How large is large?**

Statistically, $n > 30$

For $n < 30$, we assume Student t- Distribution

Gaussian Distribution vs t-Distribution

$$z = \left|\frac{x-\mu}{\sigma}\right|$$



$$t = \left|\frac{\bar{x}-\mu}{S/\sqrt{n}}\right|$$

S = sample standard of deviation
n = number of samples

Let’s do a specific example…

You have a random number of weights, of which you select a sample to weigh. The weights are the following (kg):

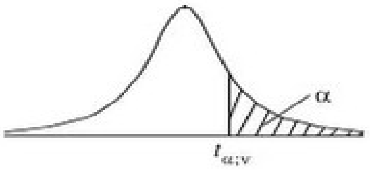
1.08	1.03	0.96	0.95	1.04
1.01	0.98	0.99	1.05	1.08
0.97	1.00	0.98	1.01	

Based on this sample, what is the 95% confidence interval for the true mean value, μ?

t-Distribution Table

Table of the Student's *t*-distribution

The table gives the values of $t_{\alpha;\nu}$ where $\Pr(T_\nu > t_{\alpha;\nu}) = \alpha$, with ν degrees of freedom



ν \ α	0.1	0.05	0.025	0.01	0.005	0.001	0.0005
1	3.078	6.314	12.076	31.821	63.657	318.310	636.620
2	1.886	2.920	4.303	6.965	9.925	22.326	31.598
3	1.638	2.353	3.182	4.541	5.841	10.213	12.924
4	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	1.319	1.714	2.069	2.500	2.807	3.485	3.767
24	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	1.310	1.697	2.042	2.457	2.750	3.385	3.646

$\nu = (n-1)$ “degrees of freedom”

α is confidence interval

$$n = 14 < 30 \ \ Student - t\ test$$
$$\nu = n - 1 = 13$$

Calculate $\bar{x}$ and S_x

$$\bar{x} = \sum_{i=1}^{14} \frac{x_i}{n} = 1.009 \text{ kg}$$

$$S_x = \sqrt{\frac{(x_1-\bar{x})^2+(x_2-\bar{x})^2+\cdots}{n-1}} = 0.4178$$

For 95% confidence level

α = 0.025 , from the table using $\nu = n - 1 = 13$

$$t = \left|\frac{\bar{x} - \mu}{S/\sqrt{n}}\right| = 2.160$$

$$\mu = 1.00 \pm 0.24$$

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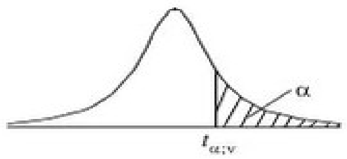

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Propagation of Error

Our goal here is to determine the total error of something that is a function of a number of independent variables

$$y\,(x_1 \pm u_1,\ x_2 \pm u_2,\ x_3 \pm u_3,\ \ldots)$$

u_n = uncertainty or error for x_n

$$\text{total error} = u_y = \sqrt{\left(\frac{\partial y}{\partial x_1} u_1\right)^2 + \left(\frac{\partial y}{\partial x_2} u_2\right)^2 + \cdots}$$

Intuitively, where does this come from?

$$\text{total error} = u_y = \sqrt{\left(\frac{\partial y}{\partial x_1} u_1\right)^2 + \left(\frac{\partial y}{\partial x_2} u_2\right)^2 + \cdots}$$

- Partial derivative is a "weighting factor" that gives us the importance of that particular term

- Each variable and corresponding error is an independent vector➔adding vectors to get magnitude

A Few More Comments

- We have focused on random errors, which leads to data being a Gaussian distribution→**Precision**
- You can also have systematic errors (**Biased**)?
- What if you have **both precision and biased errors**?

$$u_y = \sqrt{P_y^2 + B_y^2}$$