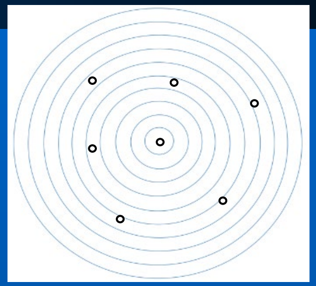
UC Berkeley
Mechanical Engineering Department

ME103:: Experimentation and Measurements

Lecture #4

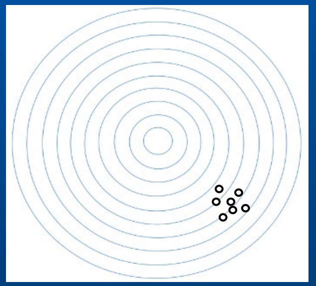
Bias and Random Error

- **Precision error = "random scatter" in the experimental results that CAN be found from repeated measurements.**
  - Different for each successive measurement but have an average value of zero



Random or Precision Error

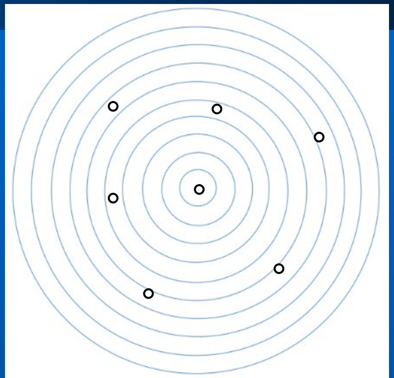
- **Bias error = Systematic errors that CANNOT be found from repeated measurements**
  - Occur the same way each time a measurement is made
    - e.g. if a scale consistently reads 5% high, the entire set of measurements will be biased +5% above the true value
  - Can be estimated by comparison of the instrument to a more accurate standard, from knowledge of how the instrument was calibrated, or from experience with that specific instrument (or type of instrument)



Systematic or Bias Error

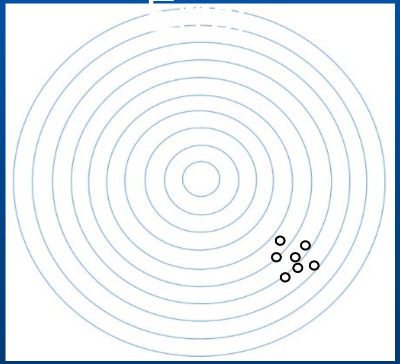
Some Causes of Bias and Random Errors

- **Precision error from:**
  - disturbances (uncompensated vibrations, temperature, pressure, etc.),
  - observation errors,
  - friction,
  - not repeating experimental procedure properly,
  - etc.



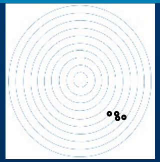
Random or Precision

- **Bias error from:**
  - calibration
  - reoccurring human error,
  - defective equipment,
  - theoretical (e.g. incorrect system model),
  - etc.



Systematic or Bias Error

- **Blunders**
  - recording data incorrectly,
  - reading scale incorrectly,
  - etc.



Blunders

Terminology: Uncertainty

- **Uncertainty: A measure of the *expected error* in measurement that combines both systematic and random errors.**

- **For sample x, if B represents an estimate of bias (systematic) errors and P represents an estimate of precision (random) errors, then the uncertainty U is usually estimated as:**

$$U_x = \left(B_x^2 + P_x^2\right)^{1/2}$$

Terminology: Uncertainty

- Most of the time, you will be dealing with random errors
- The best way to quantify random errors is to take a lot of measurements and perform statistics

How Do We Go about Quantifying Error?

- When you make measurements and take data, you will get a population of data
- Within this population, you will have a distribution about a mean value

Average or mean

$$\bar{X} = \sum_{i=1}^{n} \frac{x_i}{n}$$

x_i = data point
n = number of data points

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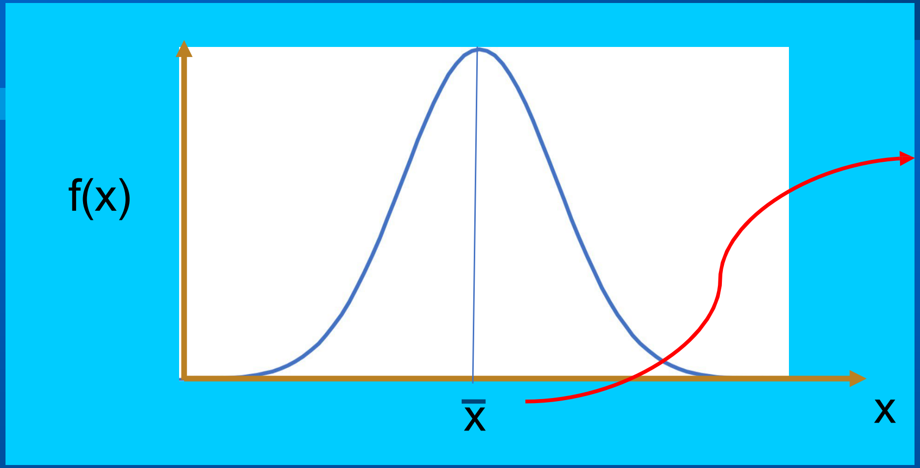
If your data are independent of one another, then the distribution about the mean is a Gaussian



Centered around the mean value

$$PDF = f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

x = individual data point
μ = TRUE value
σ = standard of deviation (gives the "spread" of the Gaussian)

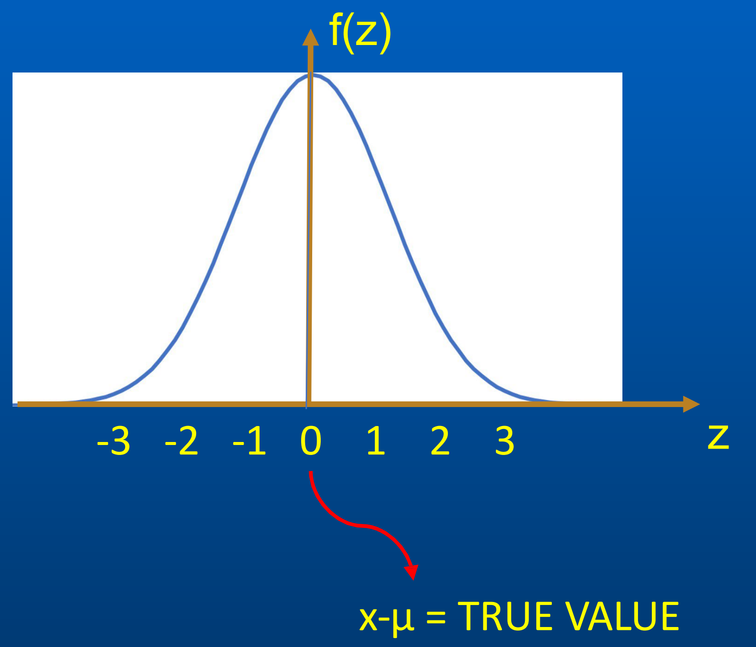
$$\sigma = \sqrt{\frac{(x_1 - \mu)^2 + (x_2 - \mu)^2 + \cdots + (x_n - \mu)^2}{n}}$$

To make life easier for us, especially when we are comparing distributions, we can **re-write the Gaussian distribution:**

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$z = \left|\frac{x-\mu}{\sigma}\right|$$

$$f(z) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{z^2}{2}}$$



By re-writing your Gaussian in terms of z, you have the following…

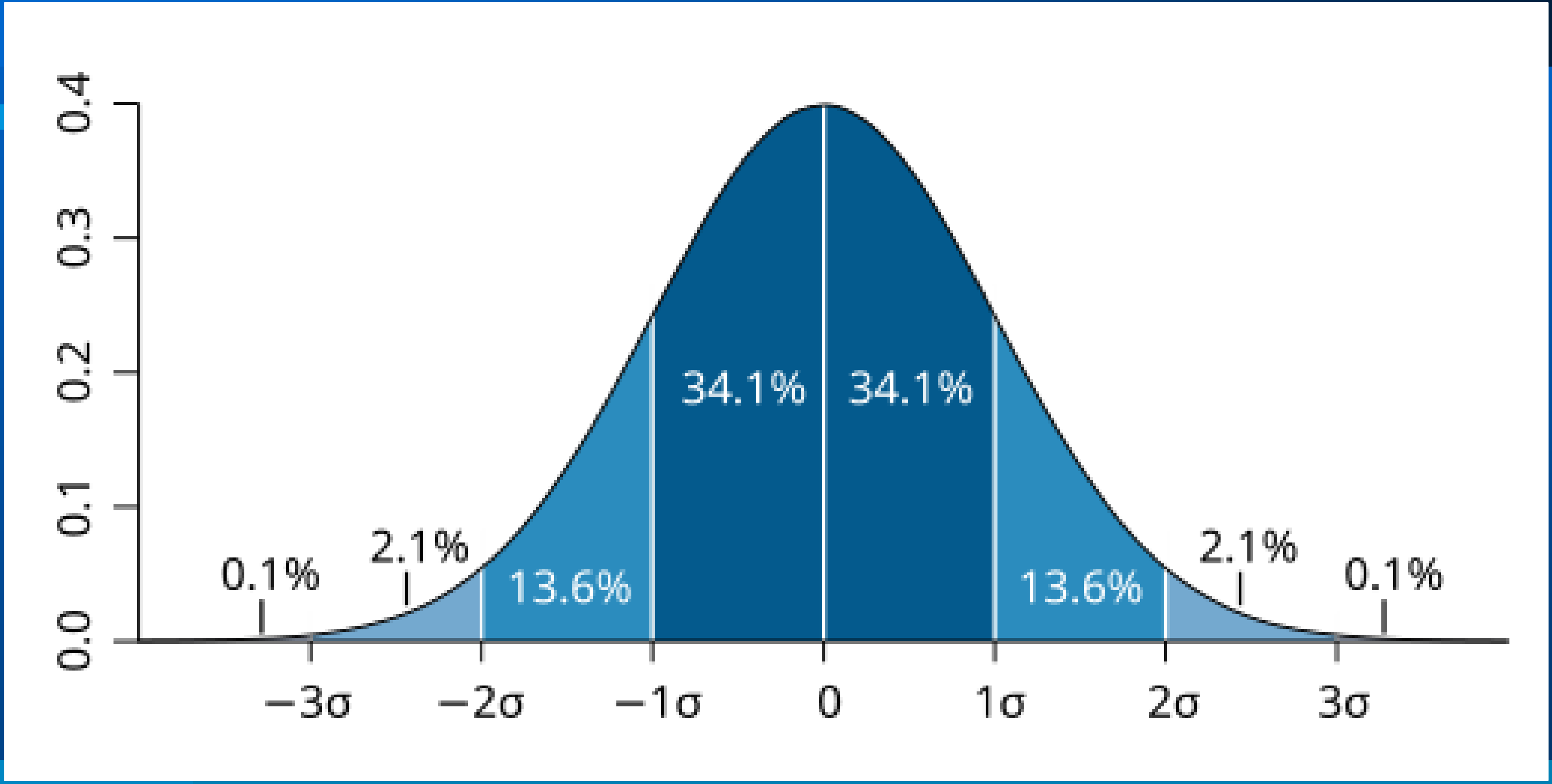
$$z = \left|\frac{x-\mu}{\sigma}\right|$$

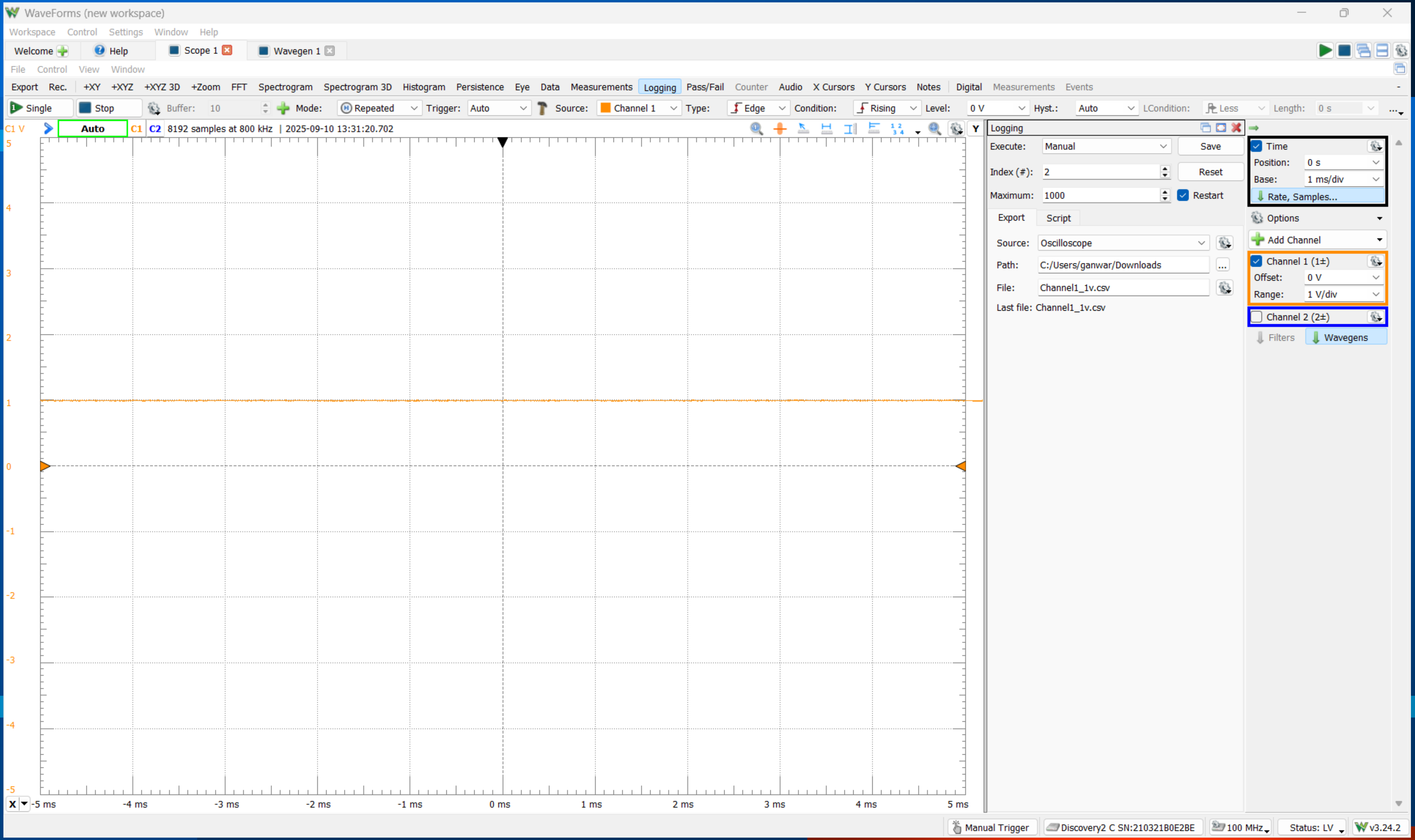
$$Z\sigma = |x-\mu|$$

$$x = \mu \pm Z\sigma \quad \text{or} \quad \mu = x \pm Z\sigma$$

$\mu = x \pm z\sigma$ is an important statement: If we do not know the true value μ, we have at least a range of what the true value is

$x = \mu \pm z\sigma$ tells us how good our data is and tells us if our data point is an outlier. It helps us decide whether our data can be "tossed" if it is σ, 2σ, 3σ away from μ


0.4
0.3
0.2
0.1
0.0
0.1%
2.1%
13.6%
34.1%
34.1%
13.6%
2.1%
0.1%
−3σ
−2σ
−1σ
0
1σ
2σ
3σ



WaveForms (new workspace)
Workspace Control Settings Window Help
Welcome Help Scope 1 Wavegen 1
File Control View Window
Export Rec. +XY +XYZ +XYZ 3D +Zoom FFT Spectrogram Spectrogram 3D Histogram Persistence Eye Data Measurements Logging Pass/Fail Counter Audio X Cursors Y Cursors Notes Digital Measurements Events
Single Stop Buffer: 10 Mode: Repeated Trigger: Auto Source: Channel 1 Type: Edge Condition: Rising Level: 0 V Hyst.: Auto LCondition: Less Length: 0 s
C1 V Auto C1 C2 8192 samples at 800 kHz | 2025-09-10 13:31:20.702
5 4 3 2 1 0 -1 -2 -3 -4 -5
-5 ms -4 ms -3 ms -2 ms -1 ms 0 ms 1 ms 2 ms 3 ms 4 ms 5 ms
Logging
Execute: Manual Save
Index (#): 2 Reset
Maximum: 1000 Restart
Export Script
Source: Oscilloscope
Path: C:/Users/ganwar/Downloads
File: Channel1_1v.csv
Last file: Channel1_1v.csv
Time
Position: 0 s
Base: 1 ms/div
Rate, Samples...
Options
Add Channel
Channel 1 (1±)
Offset: 0 V
Range: 1 V/div
Channel 2 (2±)
Filters Wavegens
Manual Trigger Discovery2 C SN:210321B0E2BE 100 MHz Status: LV v3.24.2