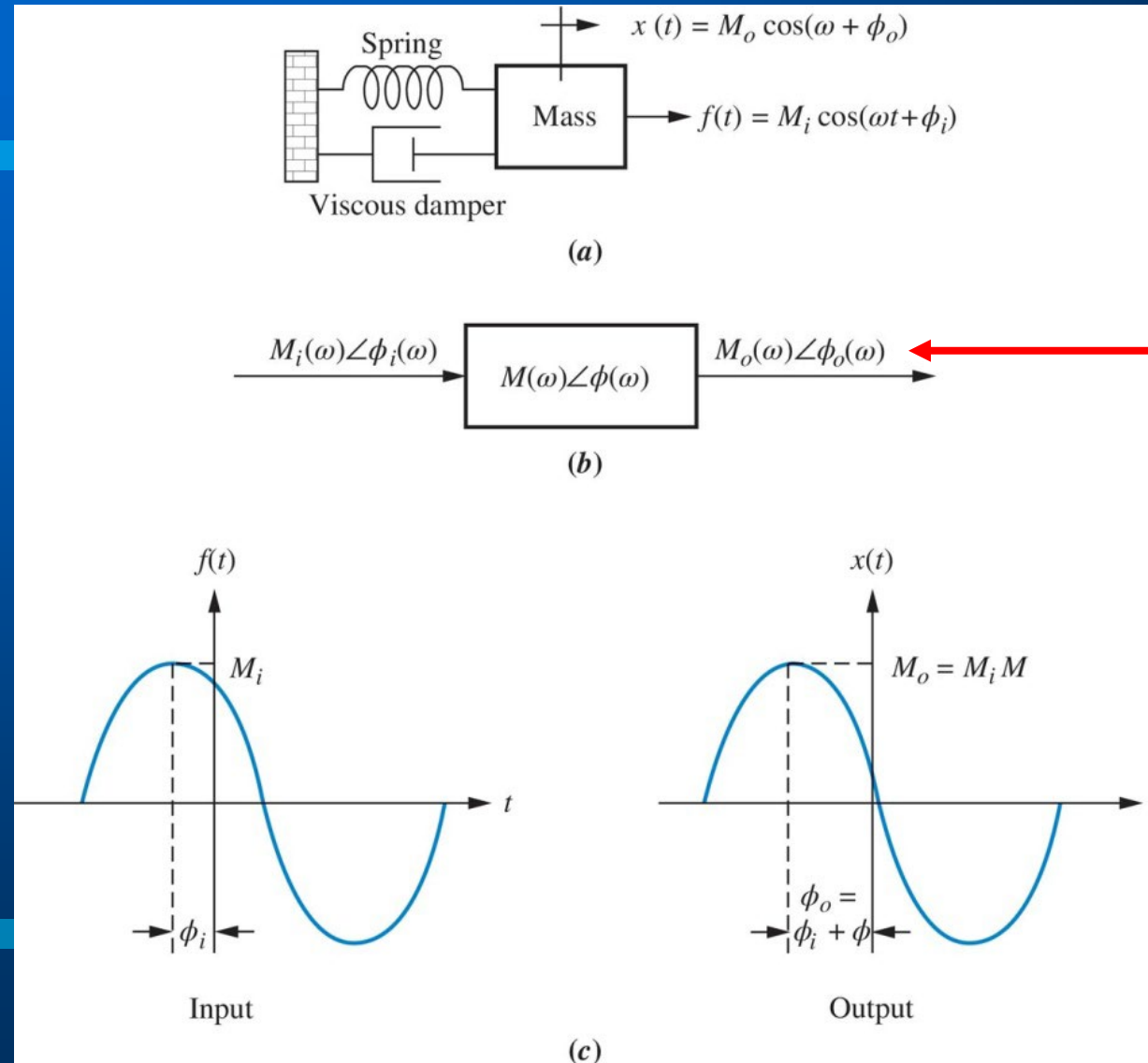


**ME103::** Experimentation and Measurements

**Lecture #10**

# Frequency Response and the Bode Plot:



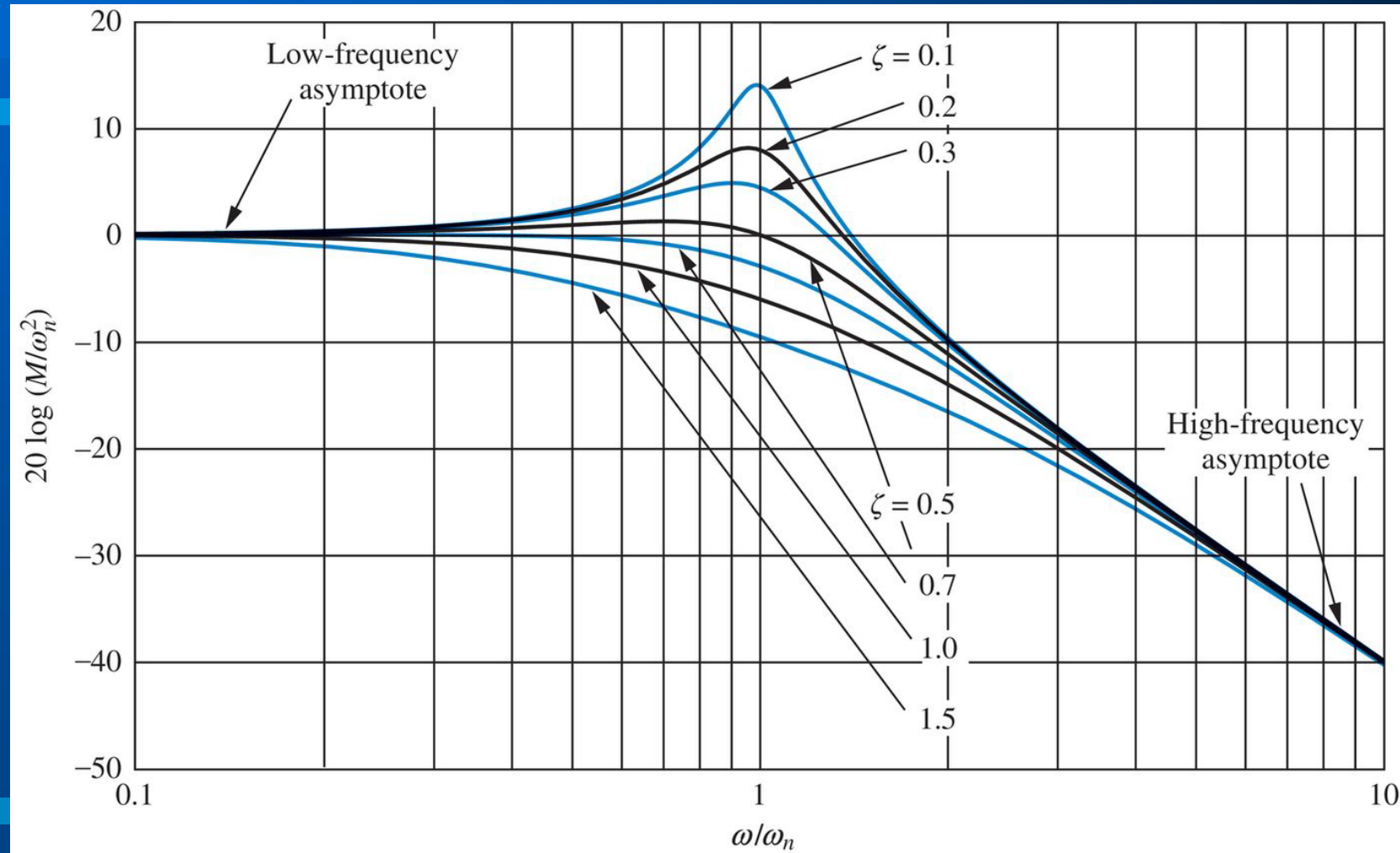
A **Bode Plot** is a graphical method used in control systems and signal processing to analyze the frequency response of a **linear time-invariant (LTI) system**.

It consists of **two separate plots**:

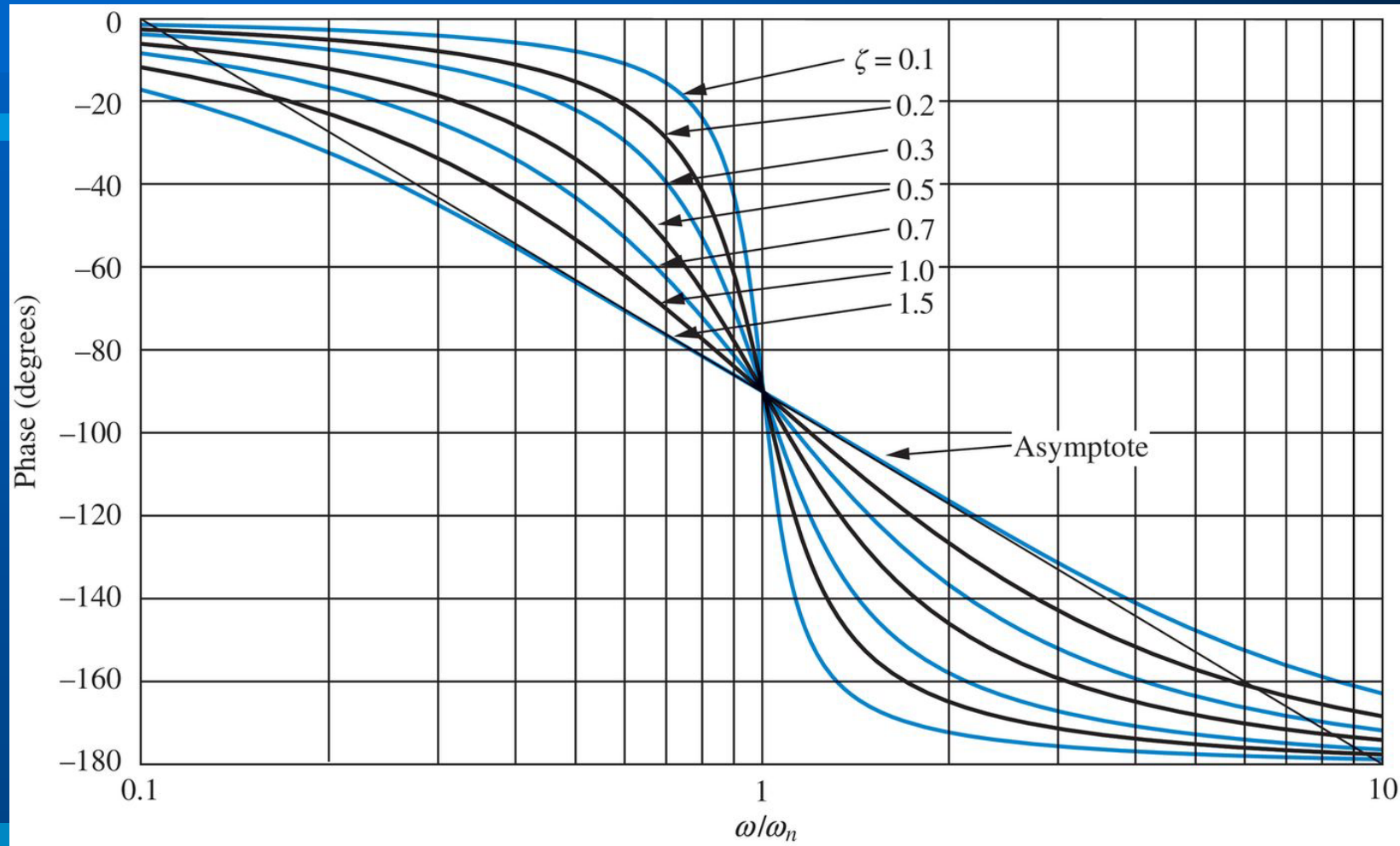
**1. Magnitude/Gain Plot** – Shows how the system gain changes with frequency. Gain is typically the ratio of the output magnitude over the input magnitude ratio. Often express in units of decibels (dB)

**2. Phase Plot** – Shows how the phase shift changes with frequency. Both plots use a **logarithmic frequency scale** (usually in rad/s).

## Bode Plot:



## Bode Plot:



Bode Plot:

$$\frac{dx(t)}{dt} = u(t)$$

$$u(t) = \cos(\omega t)$$

$$x(t) = \frac{1}{\omega} \sin(\omega t) = \frac{1}{\omega} \cos\left(\omega t - \frac{\pi}{2}\right)$$

$$M\left(\frac{x(t)}{u(t)}\right) = \left|\frac{1}{\omega}\right|$$

$$V_S(t) = V_A \cos(\omega(t) + \varphi)$$

Phasor Notations:

$$(1) \bar{V}_S = V_A \angle \varphi \quad \text{standard form} \quad (3) \bar{V}_S = V_A e^{j\varphi} \quad \text{exponential form}$$

$$(2) \bar{V}_S = a + jb \quad \text{complex form}$$

$$a = V_A \cos \varphi$$

$$b = V_A \sin \varphi$$

Converting complex to standard:

$$V_A = \sqrt{a^2 + b^2}$$

$$\varphi = \tan^{-1}\left(\frac{b}{a}\right)$$

Example:

Express  $V_s(t) = 3 \sin(100(t) + 40^\circ)$  in standard and complex phasor form

$$V_s(t) = 3 \cos(100(t) + 40^\circ - 90^\circ)$$

$$V_s(t) = 3 \angle -50^\circ \quad \text{standard form}$$

$$a = 3 \cos(-50^\circ) = 1.928$$

$$b = 3 \sin(-50^\circ) = 2.298$$

$$V_s(t) = 1.928 - j2.298 \quad \text{complex form}$$

## Bode Plot:

```
MATLAB R2020b - academic use
HOME PLOTS APPS
C:\Users\ms
Command Window
>> sys1 = tf([1],[1])

sys1 =

    1

Static gain.

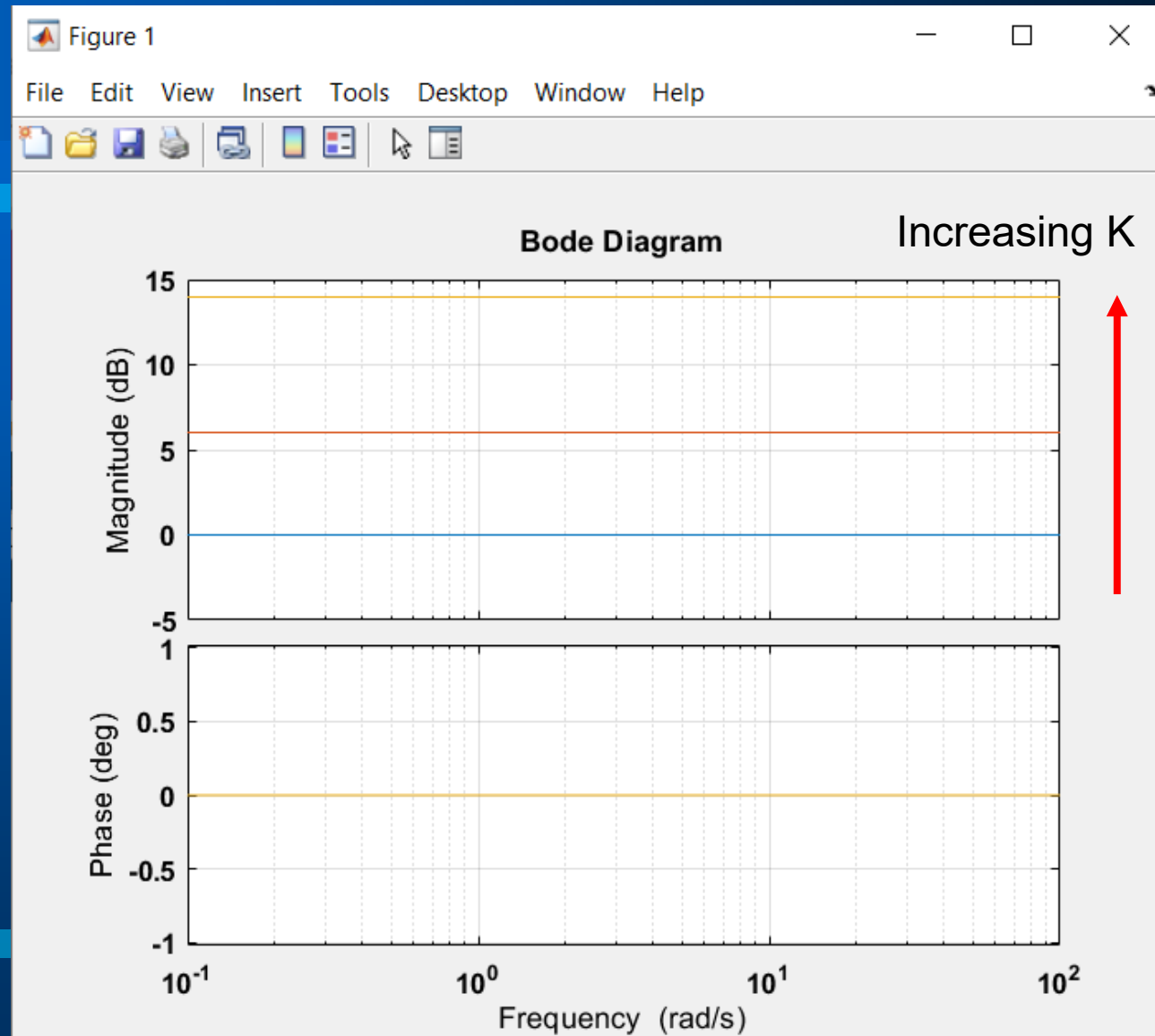
>> bode(sys1)
>> hold
Current plot held
>> bode(2*sys1)
>> bode(5*sys1))
```

 $|K|$ 

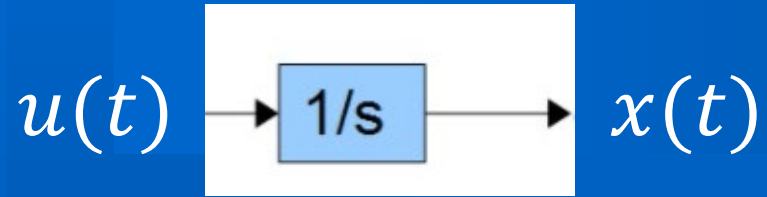
$$20\log|1| = 0$$

$$20\log|2| \approx 6.02$$

$$20\log|5| \approx 13.98$$



Bode Plot:



$$x(t) = \int_{0^-}^t u(t) dt$$

$$\frac{dx(t)}{dt} = u(t)$$

$$u(t) = \cos(\omega t)$$

$$x(t) = \frac{1}{\omega} \sin(\omega t) = \frac{1}{\omega} \cos\left(\omega t - \frac{\pi}{2}\right)$$

## Bode Plot:

Command Window

```
>> sys1 = tf([1],[1 0])
```

```
sys1 =
```

$$\frac{1}{s}$$

```
Continuous-time transfer function.
```

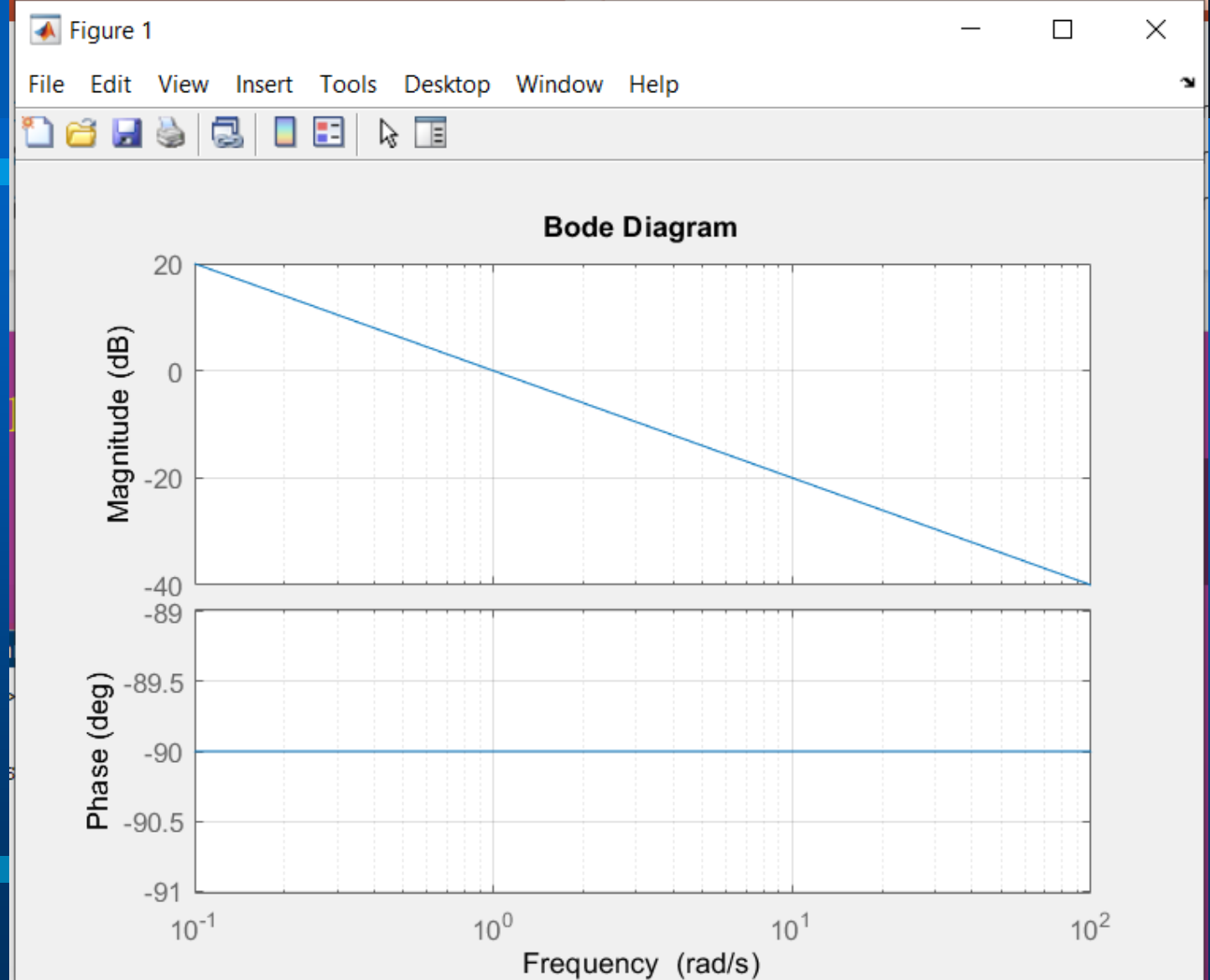
```
>> bode(sys1)
```

```
fx >> |
```

$$j\omega \rightarrow s$$

$$\frac{O(j\omega)}{I(j\omega)} = \left| \frac{1}{j\omega} \right|$$

$$M(j\omega) = 0 - \frac{1}{\omega}j$$



$$V_S(t) = V_A \cos(\omega(t) + \varphi)$$

Phasor Notations:

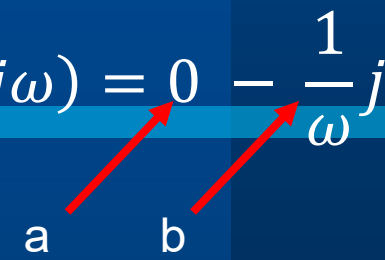
$$(1) \bar{V}_S = V_A \angle \varphi \quad \text{standard form} \quad (3) \bar{V}_S = V_A e^{j\varphi} \quad \text{exponential form}$$

$$(2) \bar{V}_S = a + jb \quad \text{complex form}$$

$$a = V_A \cos \varphi$$

$$b = V_A \sin \varphi$$

Converting complex to standard:

$$M(j\omega) = 0 - \frac{1}{\omega}j$$


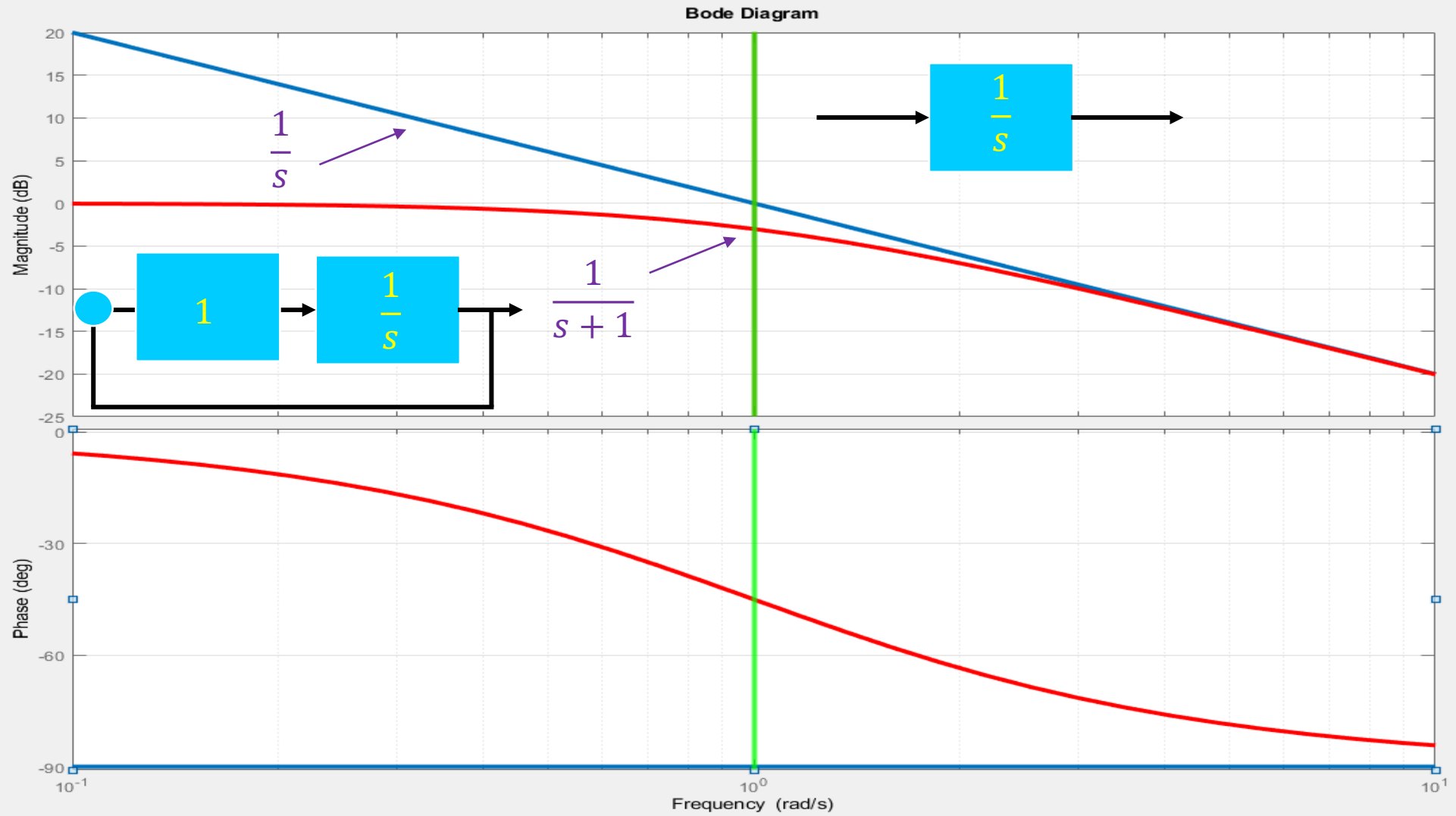
a

b

$$V_A = \sqrt{a^2 + b^2} = \sqrt{0^2 + \left(-\frac{1}{\omega}j\right)^2} = j \left| \frac{1}{\omega} \right|$$

$$\varphi = \tan^{-1}\left(\frac{b}{a}\right) = \tan^{-1}\frac{-\frac{1}{\omega}}{0} = -\frac{\pi}{2}$$

# Using Bode Plot to design (Frequency Shaping):

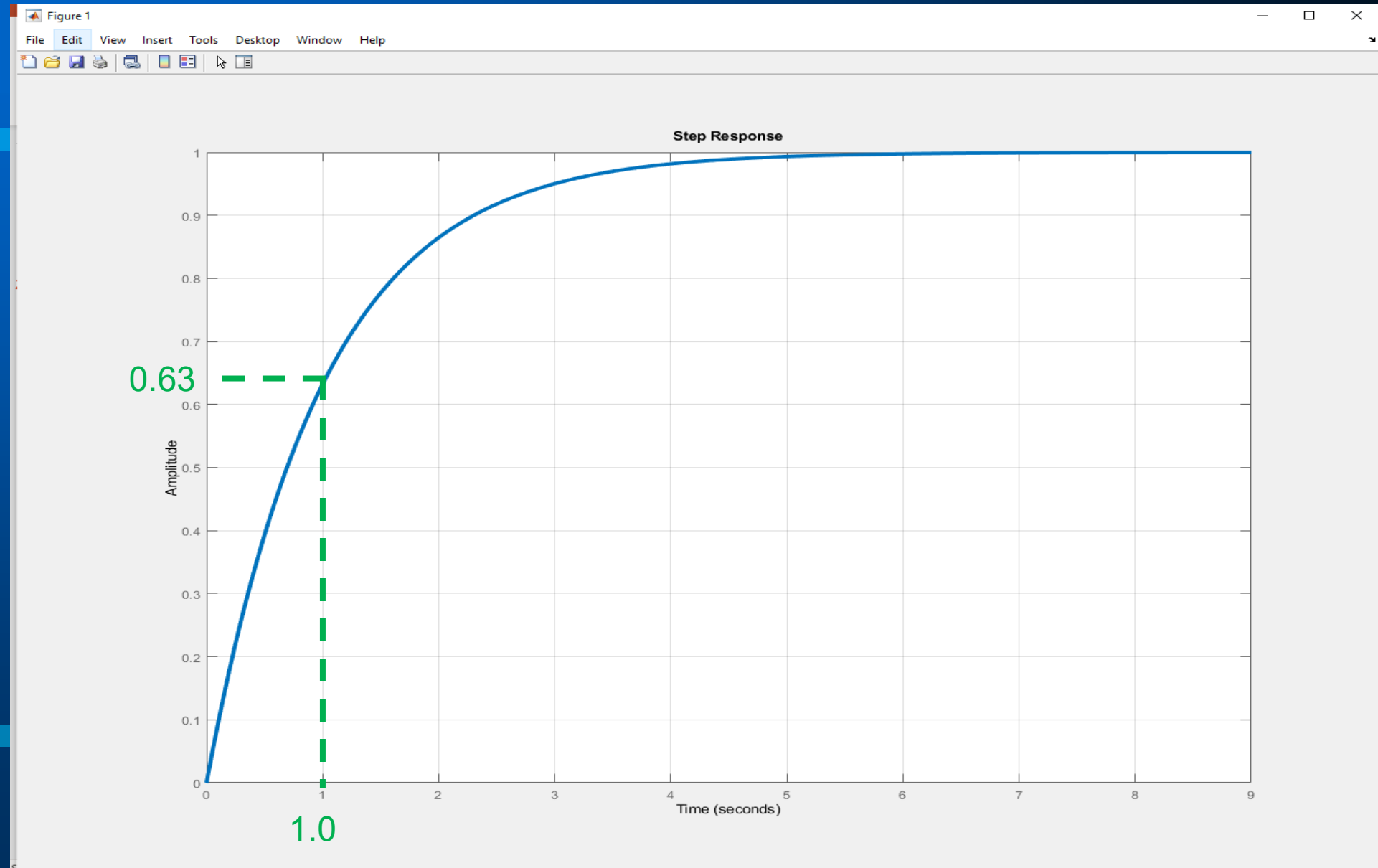


# Using Bode Plot to design (Frequency Shaping):

Closed Loop TF:

$$\frac{1}{s + 1}$$

$$c(t) = 1 - e^{-t}$$



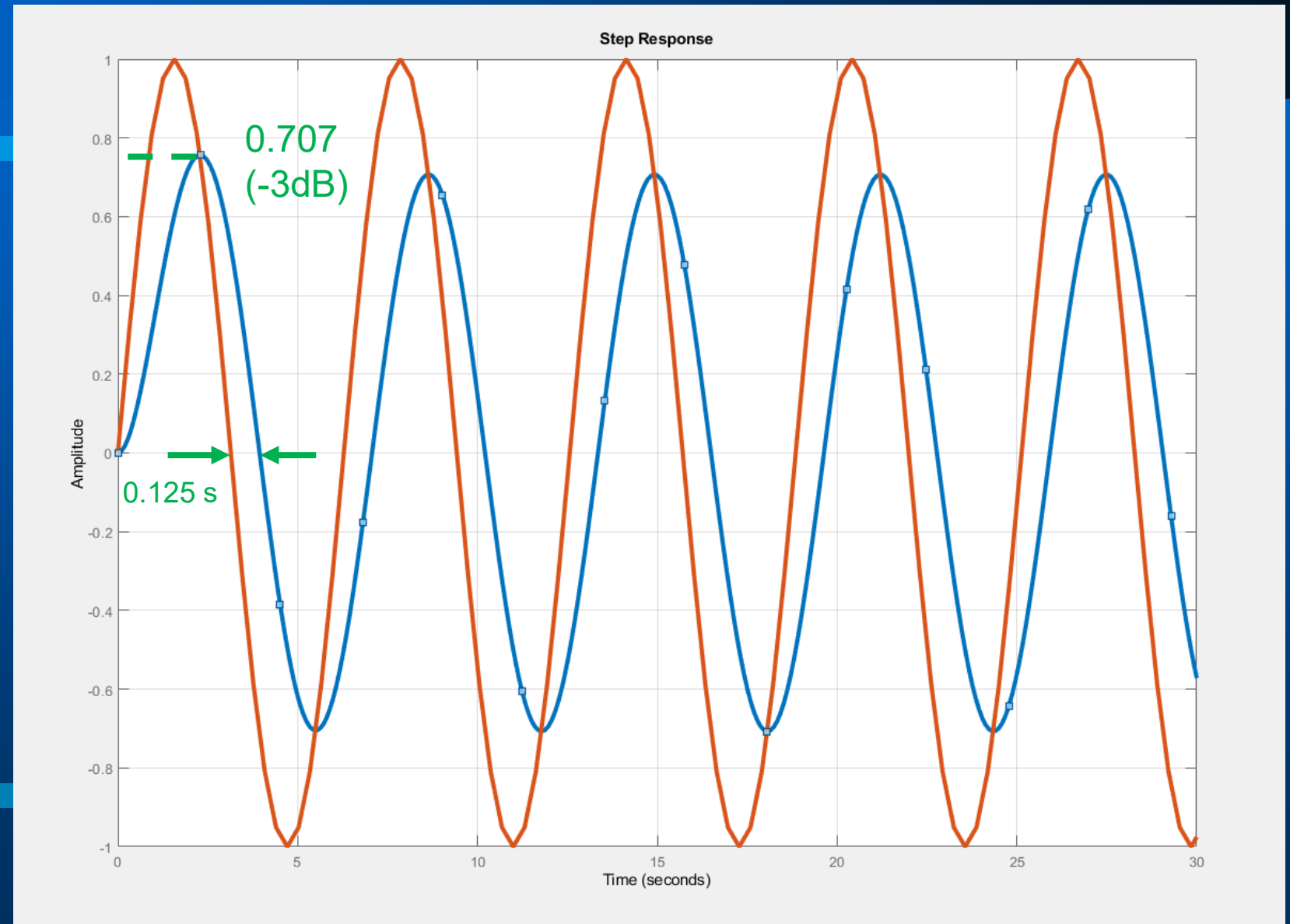
## Using Bode Plot to design (Frequency Shaping):

@ $\omega = 1 \text{ rad/s}$

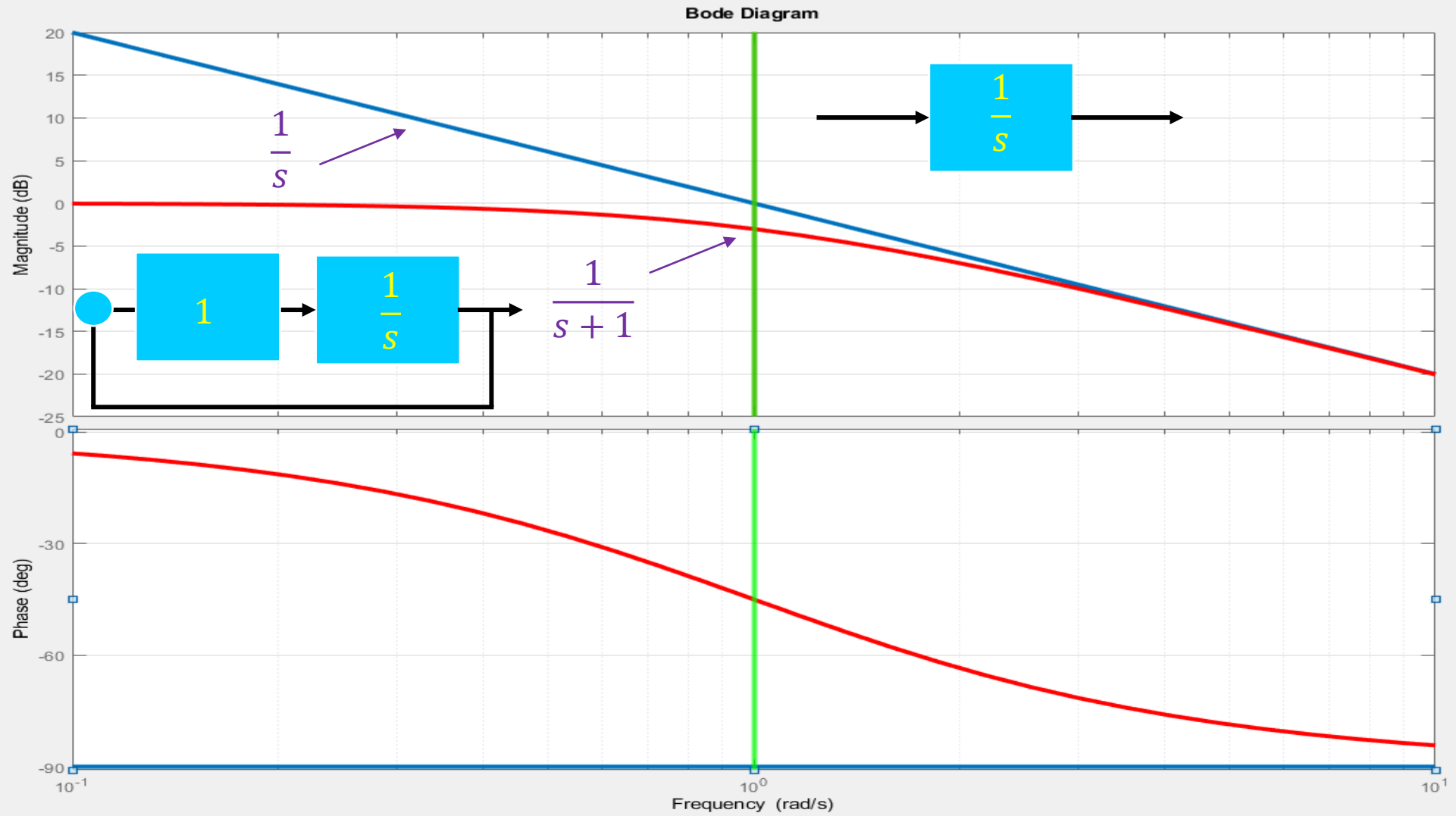
Open loop gain = 1

Closed loop gain = -3dB

Closed loop phase =  $-45^\circ$



# Using Bode Plot to design (Frequency Shaping):



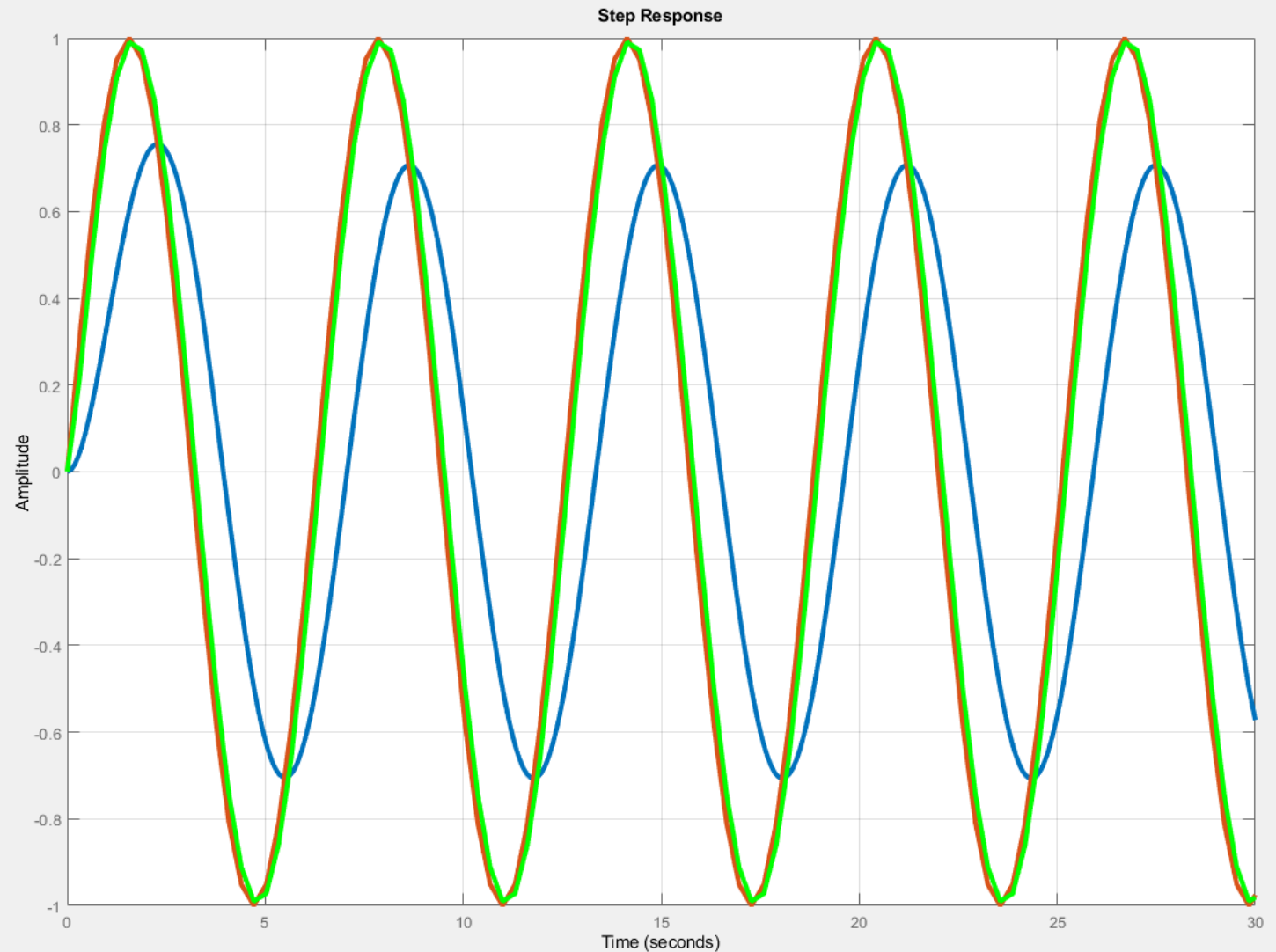
## Using Bode Plot to design (Frequency Shaping):

@ $\omega = 1 \text{ rad/s}$

Open loop gain = 10 = 1dB

Closed loop gain = -3dB

Closed loop phase =  $45^\circ$



## Bode Plot:

### Magnitude (Standard Form)

$$|G(j\omega)| = \frac{1/m}{\sqrt{(\omega_n^2 - \omega^2)^2 + (2\zeta\omega_n\omega)^2}}$$

