ME 103 Discussion 4

Week of 2/9

Announcements

- Don’t come to lab next week. There is no lab, only a pre-lab assignment. Do it at home.
- Take the time to understand the upcoming lab, making the physical lab (we will do that the week after) easier
- Estimated time to complete ~ 3 hours w effort

Hypothesis Testing

- A way to test a claim using data from a sample
- We assume the claim is true, then check if the evidence disagrees
  - We need strong enough evidence to reject the claim.
- The claim we are assuming to be true is the **null hypothesis H_0** and the alternative is called the **alternative hypothesis H_a**

One Sided vs. Two Sided

- A one-sided (one-tailed) test checks for a difference in **one direction only.**
    - E.g.: “bigger than” or “smaller than”
- A two-sided (two-tailed) test checks for a difference in **either direction.**
    - E.g. “different from” or simply “not equal to”

Testing a Hypothesis Part 1

- Step 1: Define Hypothesis
  - Assuming the claim is **true**
    - H_0 : "The average salt content is less than or equal to 2.0g."
  - Assuming the claim is **false**
    - H_a : "The mean salt content is greater than 2.0 g."
- Step 2: Picking a test
  - We are also given n>30, population variance, significance level
  - We will use a **z-test** to test our hypothesis.
  - Think: "assuming H_0 is true, how realistic is it for us to have a sample mean of 2.2g?"
- Step 3: Calculate test statistic
  - You are familiar with z score of a single data point–but now need the z score of a **sample mean**, which is a different formula.

this is a **one sided test**, we only care if it is greater than 2.0g!

Z-TEST

- Formula to find the value of Z (z-test) Is:

$$Z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$$

- $\bar{x}$ = mean of sample
- μ_0 = mean of population
- σ = standard deviation of population
- n = no. of observations

Testing a Hypothesis Part 2

- Step 4: Find Critical Value based on Significance Level
  - Sig Level: Essentially the amount of error you would accept. (typical: 5% → = 0.05)
  - If significance level 5% for a **one tailed test,** you are looking for the z score that contains **95% of the data** (0.95 on the table)
- Step 5: Compare z crit to z calculated
  - Since we are looking for if mean salt is **greater** than 2.0, we reject our null hypothesis if our calculated value is greater than our critical value (i.e. our sample falls higher than where we would expect (1-α)% of values to fall.
  - If Z_calculated > Z_critical:
    - Reject H_0 (reject the claim)
  - If Z_calculated ≤ Z_critical:
    - Fail to reject H_0 (accept the claim)
- Step 6: Conclusion
  - *"There's statistically significant evidence that the true mean salt content **does/does not** exceed 2.0g per serving; Null Hypothesis claim **is/is not** supported at the (1-α)% significance level."*

> Note: take the time to digest this process, **you are expected** to follow this format (especially the conclusion wording) on assignments and exams.

One sample vs. Two sample

- Two sample uses two different population groups
- “Online group vs. in person group having **different** average scores”
- H_0: $\bar{x}_{in\ person} - \bar{x}_{online} = 0$
  - Two tailed, two sample

Z- Test Formula

$$Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

One-Sample T-Test

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

$\bar{x}$ = obersved mean of the sample
μ = assumed mean
s = standard deviation
n = sample size

Two-Sample T-Test

$$t = \frac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$\bar{x}_1$ = observed mean of 1st sample
$\bar{x}_2$ = observed mean of 2nd sample
s_1 = standard deviation of 1st sample
s_2 = standard deviation of 2nd sample
n_1 = sample size of 1st sample
n_2 = sample size of 2nd sample

Final Caveat: Equal vs. Unequal variances

"You may assume equal variance"

- Both populations come a larger population with the same variance.

"You may not assume equal variance"

- Keep standard deviations separate.

One Sample t Test

$$t_{cal} = \frac{(\bar{x} - \mu)}{s/\sqrt{n}}$$

Two Sample t Tests

QG

❖ Is variance for two samples equal?

Yes

$$t_{cal} = \frac{(\bar{x}_1 - \bar{x}_2)}{s_p\sqrt{\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

$$df = n_1 + n_2 - 2$$

No

$$t_{cal} = \frac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)}}$$

$$df = \frac{\left[\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right]^2}{\frac{(s_1^2/n_1)^2}{(n_1 - 1)} + \frac{(s_2^2/n_2)^2}{(n_2 - 1)}}$$

Same process afterwards: use calculated t and compare it to the crit t value (based on t table and significance level) and reject or accept null Hypothesis.

Note: this is only relevant for t-tests!

Non-Parametric Tests

- What if population size is *not* normal *and* sample size is small?
  ⇒ Non-Parametric Testing
- In the case of Problem 2, n = 4, looking for median instead of mean.
- Non-parametric tests are *distribution free* tests used for ordinal, nominal data etc.
- **The ones you need to know:**
  - Wilcoxon Rank-Sum Test (Mann-Whitney U test)
  - Wilcoxon Signed-Rank Test
- Question: are the populations being tested *the same or different populations?*
  - Mann-Whitney U
    - used for: Independent groups - different customers rating Company A vs Company B.
  - Wilcoxon Signed Rank
    - Used for: Paired/matched data - same customers rating BOTH companies.

Full list of non-parametric tests is on the bCourses Formularry (may be useful for your projects).

Wilcoxon-rank Sum Test

- List all the scores Lowest to highest
  - 9,10,11,15,17,19,20,20
  - Assign each score an ascending rank, 9 is the lowest rank and 20 is the highest:
  - 1(B), 2(A), 3(A), 4(B), 5(A), 6(B), 7.5(B), 7.5(A)
    - **Tie Rule: there is a tie between rank 7 and 8, it goes to A and B equally so they each get 7.5.**
  - Sum the ranks of company A: T_A = 2+3+5+...
  - Sum the ranks of company B: T_B = 1+4+6+...

Company	Satisfaction
B	9
A	10
A	11
B	15
A	17
B	19
B	20
A	20

Wilcoxon-rank Sum Test

Plug in look for TL and TU for n1=4 and n2 = 4. If your summed ranks T_A or T_B fall within the range of T_L to T_U, then you **accept the null hypothesis.**

n_2 \ n_1	3		4		5		6		7		8		9		10	
	T_L	T_U	T_L	T_U	T_L	T_U	T_L	T_U	T_L	T_U	T_L	T_U	T_L	T_U	T_L	T_U
3	5	16	6	18	6	21	7	23	7	26	8	28	8	31	9	33
4	6	18	11	25	12	28	12	32	13	35	14	38	15	41	16	44
5	6	21	12	28	18	37	19	41	20	45	21	49	22	53	24	56
6	7	23	12	32	19	41	26	52	28	56	29	61	31	65	32	70
7	7	26	13	35	20	45	28	56	37	68	39	73	41	78	43	83
8	8	28	14	38	21	49	29	61	39	73	49	87	51	93	54	98
9	8	31	15	41	22	53	31	65	41	78	51	93	63	108	66	114
10	9	33	16	44	24	56	32	70	43	83	54	98	66	114	79	131

Next Week:

- Introduction to the frequency domain!
  - Frequency Resolution, Nyquist Criterion
  - Aliasing