ME 103 Discussion 6

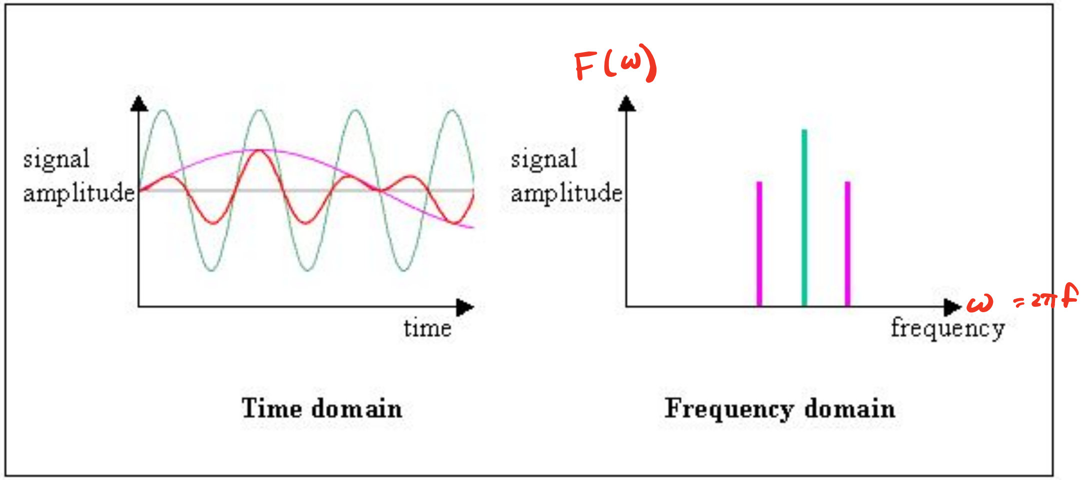
Week of 2/16

Review of Concepts

Time Domain to Frequency Domain

- The **Time Domain** is a familiar way to represent signals.
    - Voltage & Current represented as a function of time (sinusoidal)
- The **Frequency Domain** represents signals as a magnitude and phase as a function of frequency
- Same information, different format.

↳ gives us insight as to what lies in our data



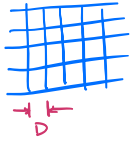
Why Does Spectral Analysis Matter in ME?

range of freq. ⇒ want to know how much energy of the signal is located at each freq.

- *Spectrum :* the distribution of a signal as a function of frequency
- *Frequency* : most often *temporal* frequency, also often *spatial* frequency

time varying signal

$\frac{1}{T}$ ← period



$\frac{1}{D}$ ← D: spacing b/w pixels in image

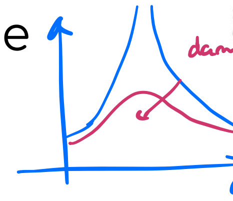
- Reasons for caring about spectral analysis:
  - Resonance happens at specific frequencies!
  - Controllers and circuits – have a limit to how quickly they can switch – because of parasitics
  - Want to control which frequencies are emphasized in a signal (e.g. audio processing, Shazam!)
  - Want to compress a signal to save space or data transmission bandwidth (...HW3 → MP3 Lossy Audio Compression!)

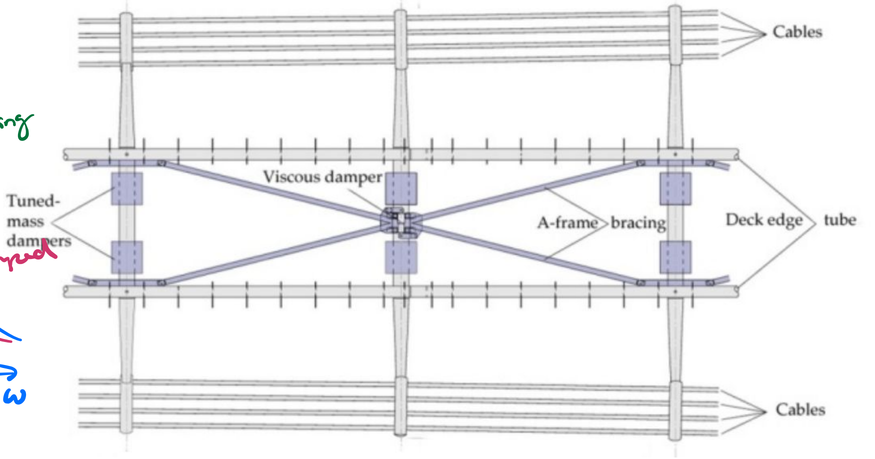
Example 1: Mechanical Vibrations

- Resonances happen at specific, design-dependent frequencies
- Resonance occurs in
    - Bridges
    - Wave energy/marine structures
    - Aircraft Structures
- Resonances can be suppressed by
    - *Damping* the structure
    - Designing resonance frequencies to not coincide with operation/excitation frequencies

resistance to motion
⇒ sucks energy out of system before it gets going

Modal Analysis

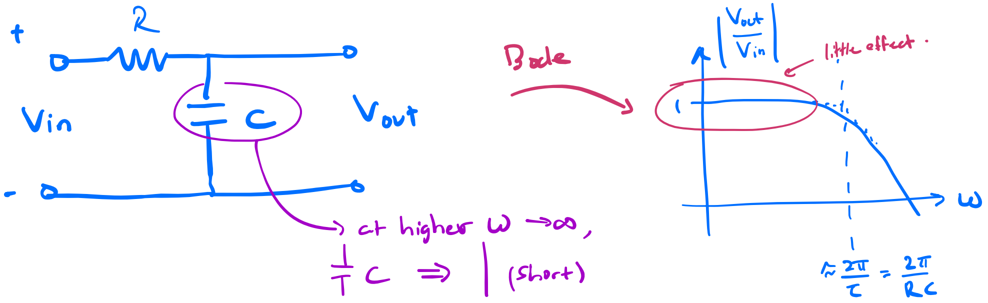




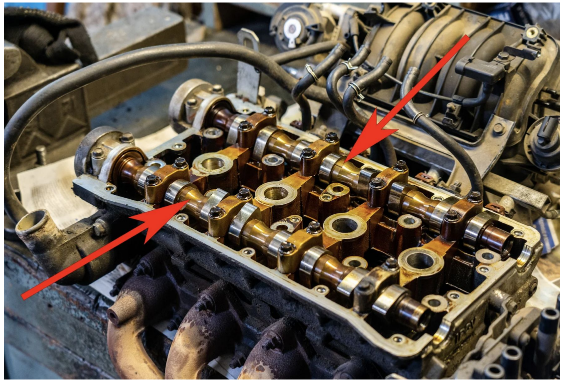
Example 2: Controllers + Circuits

Control Systems have a limit to how quickly they can change their output

- Because of electrical circuit parasitics → if you had 2 wires next to each other, there would be EM coupling i.e. capacitance (parasitic) ⇒ inherent resistance



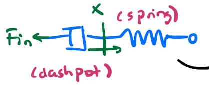
- Because of Mechanical Losses



→ CAM Lab, if cam shaft runs faster, the valves won't have time to close as CAM passes.

↳ TRUE! if there is some dissipative force going on in a system.

Analogous to



→ depends on freq!

Example 3: Signal Transmission

- Radio signal transmission uses specific parts of the EM spectrum
- Usage of the RF spectrum is regulated, and systems need to operate within allocation *bandwidth* (range of frequencies)

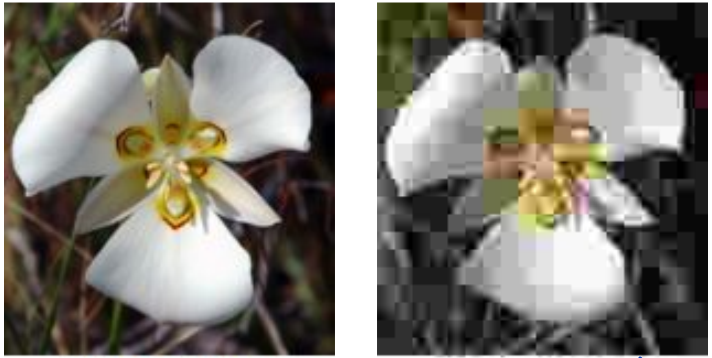
larger bandwith ⇒ more data into channel of transmission

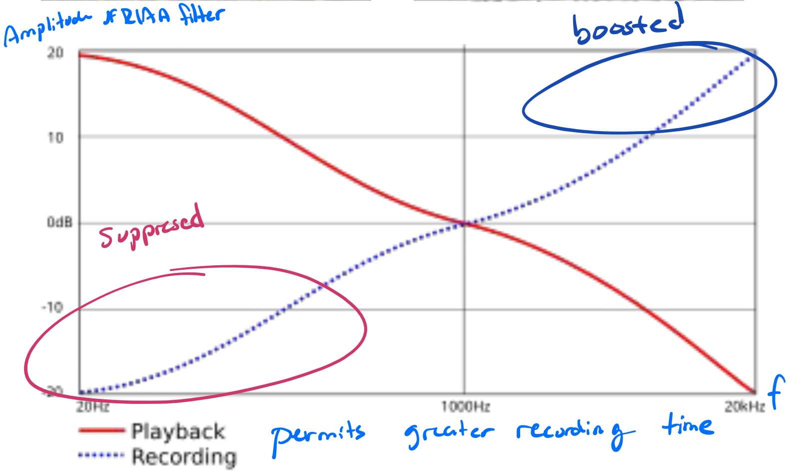
Example 4: Audio and Visual Signals

- Modifying sounds when producing music or EQs in a CDJ (bass, treble)
- Compensating for characteristics of audio reproduction equipment
  - e.g. RIAA filter in vinyl players

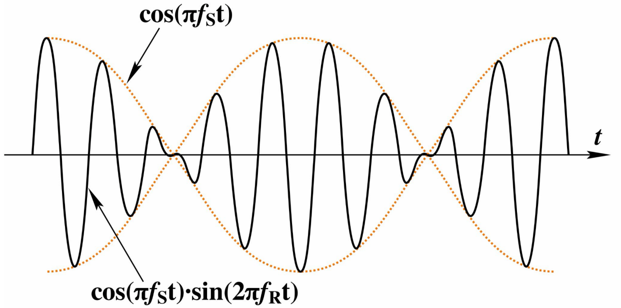
standard freq. response

- Compression of audio, photos, and video [relies on discarding freq components with little strength/information]
- Image processing: edge sharpening and denoising [rely on manipulating freq. content of the image]





Beating Between Signals



Small

- Superimpose 2 waves that differ by $\Delta\omega$ in frequency
- Beating = periodic fluctuation in amplitude that occurs when two waves of slightly different frequencies interfere with each other

$y = A\sin(\omega_0 t) + A\sin((\omega_0 + \Delta\omega)t)$ use $\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$

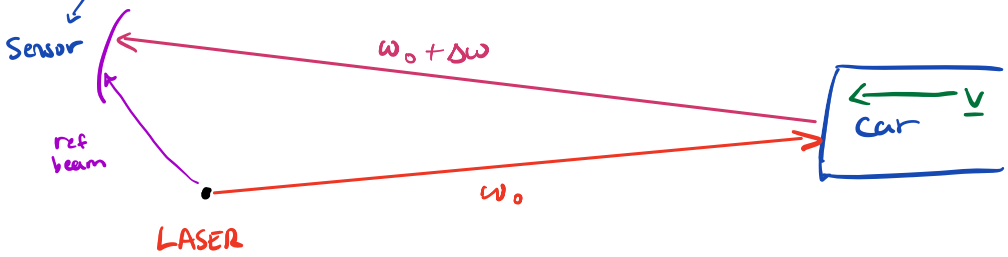
$= 2A\underbrace{\cos\left(\frac{\Delta\omega}{2}t\right)}_{\text{Low } \omega}\underbrace{\sin\left(\frac{2\omega_0+\Delta\omega}{2}t\right)}_{\text{high } \omega}$

- Very useful: can make the unmeasurable measurable
- Example: Doppler effect in LiDAR + Doppler Vibrometry

if a signal oscillates too fast,
if we make $\Delta\omega$ small, we can detect it.

(time of flight LiDAR)

add photons of both vehicle + laser ⇒ beat together that has $\Delta\omega$ low enough



$\Delta\omega = |f_1 - f_2|$

- Let's do it an actual example using Audacity and sounds!

Heterodyning

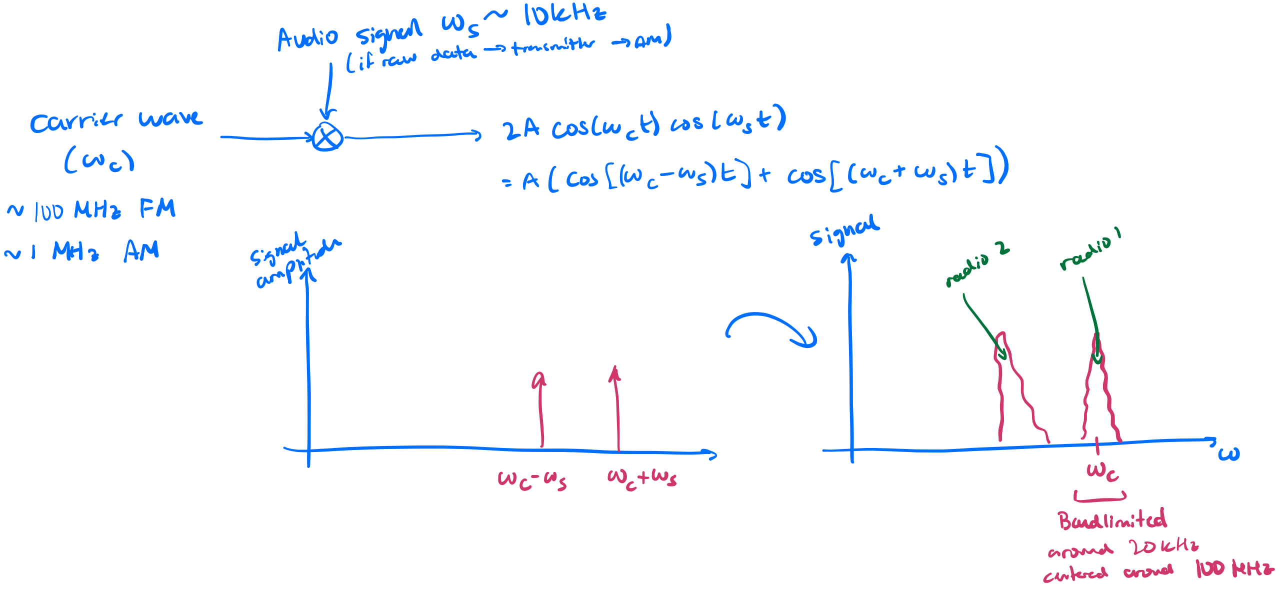
→ like opposite of beating

mapping low freq signal onto higher freq. carrier wave

- Shifting a signal to a different central frequency (i.e. to aid in transmission)

→ Allocation/ attenuation by atmosphere

  - *Shifts frequencies you can't hear into the range you can hear*
- Example: modulating a carrier wave with a radio signal



$$2A\cos(\omega_c t)\cos(\omega_s t)$$

$$= A\left(\cos[(\omega_c-\omega_s)t] + \cos[(\omega_c+\omega_s)t]\right)$$

Continuous-Time Fourier Transform (CTFT)

- [How to find our frequency content of a signal?] If the signal (doesn't need to be periodic) is continuous-time, use the Fourier Transform

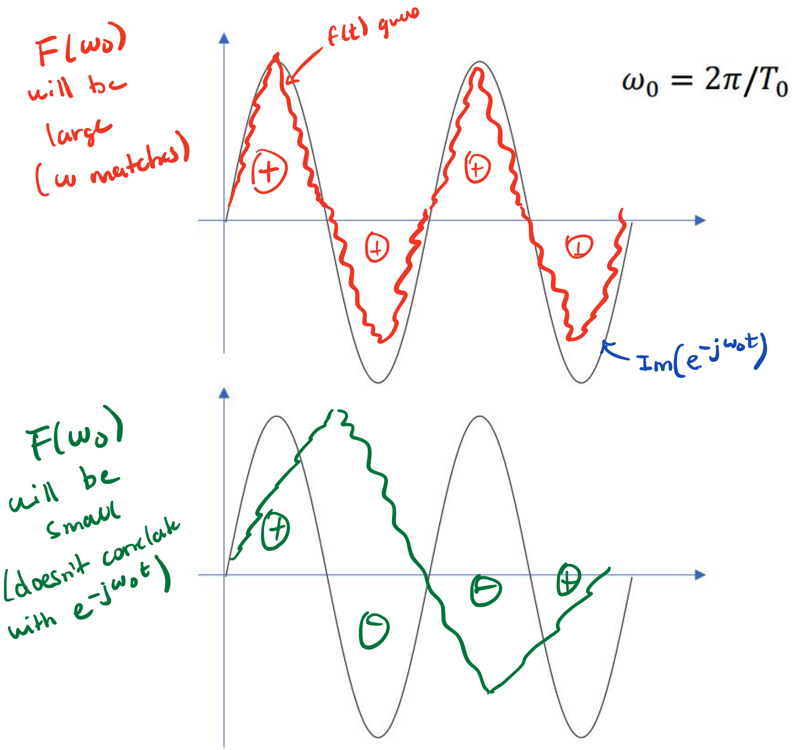
$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\, e^{j\omega t}\, d\omega \xleftrightarrow{\mathcal{F}} X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

- However:
  - we don’t actually measure signals continuously anymore
  - We sample periodically instead (couldn’t really integrate over infinite time anyways)

What is the CTFT really *doing*?

- Consider the contribution to the FT of a signal f(t) at a specific freq ω_0
- By integrating the product of the signal and a “probing” sinusoidal wave at ω, we find out if the signal has significant content at that freq. [If it does, then a high +ve value of f(t) correspond with high +ve valueof the “probing” sinusoid, and vice versa, and the resulting integral (the FT at ω) is large.]
- Masking the signal with one sine wave at a time to find out which frequencies of sine wave the signal correlates closely with.

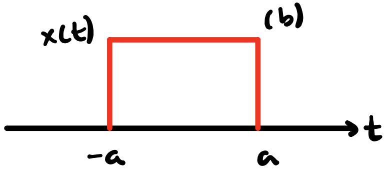
$$F(\omega_0) = A\int_{-\infty}^{\infty} f(t)\mathrm{e}^{-j\omega_0 t}\mathrm{d}t$$

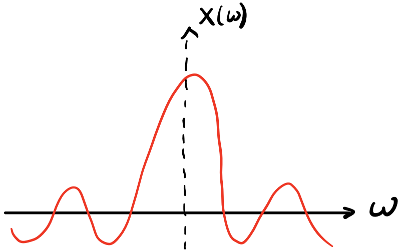


Example 5: Rectangular Pulse

Determine *X*, the CTFT of the signal *x* given as $x(t) = \begin{cases} b & |t| \leq a \\ 0 & \text{elsewhere.} \end{cases}$

The parameters *a* and *b* have appropriately-chosen +ve values.





$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-i\omega t}\, dt$$

$$= \int_{-a}^{a} b\, e^{-i\omega t}\, dt$$

$$= b \int_{-a}^{a} e^{-i\omega t}\, dt$$

$$= b \left[\frac{e^{-i\omega t}}{-i\omega} \right]_{-a}^{a}$$

$$= b \left[\frac{e^{-i\omega a}}{-i\omega} - \frac{e^{i\omega a}}{-i\omega} \right]$$

$$= -\frac{b}{i\omega} \left[e^{-i\omega a} - e^{i\omega a} \right]$$

$$= -\frac{b(-2\cancel{i})}{\cancel{i}\omega} \left(\frac{e^{i\omega a} - e^{-i\omega a}}{2i} \right)$$

$$\boxed{\begin{aligned} X(\omega) &= \frac{2b}{\omega} \sin(a\omega) \\ &= 2ab\, \mathrm{sinc}(\omega a) \end{aligned}}$$

Discrete Fourier Transform (DFT)

- If the signal is DT (need not be periodic), use the DFT

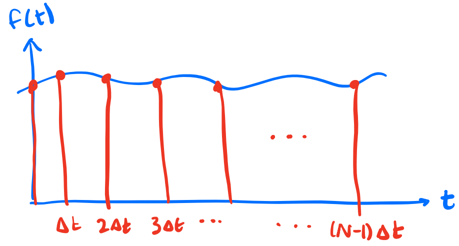
Δt = Sampling period , N = # samples

$$F(\omega_k) = A \sum_{n=0}^{N-1} f(n\,\Delta t)\, e^{-j\omega_k n\,\Delta t}$$

discrete Freq.

index of Sample

$$\omega_k = \frac{2\pi k}{N\Delta t}\ ,\quad k=0,\dots,N-1\ ,\quad A = \frac{1}{2\pi N} \text{ or } \frac{1}{2\pi}$$



- With the DFT, you can *reconstruct* the original signal with the inverse DFT

$$f(n\,\Delta t) = \frac{1}{N}\sum_{k=0}^{N-1} F(\omega_k)\, e^{2\pi jkn/N}$$

n = index in time $f(n\Delta t) = f_n$

k = index in freq. ω_k

Notes:

- Reconstruction is a summation of sinusoids
- Therefore it is periodic, with period $N\Delta t$ (perhaps unlike the original signal)
- Picking a large enough N and small enough Δt matters, to represent the signal accurately enough

Fast Fourier Transform (FFT)

- The FFT is an efficient way of computing the DFT (Butterfly Transform)

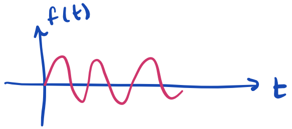
$$F(\omega_k) = A\sum_{n=0}^{N-1} f(n\Delta t)\, e^{-j\omega_k n \Delta t}$$

$$X[k] = \underbrace{\sum_{r=0}^{\frac{N}{2}-1} g(r) W_{\frac{N}{2}}^{kr}}_{G(k),\ \text{the } \frac{N}{2}\text{-pt DFT of } \{g(n)\}_{n=0}^{N/2-1}} + W_N^k \underbrace{\sum_{r=0}^{\frac{N}{2}-1} h(r) W_{\frac{N}{2}}^{kr}}_{H(k),\ \text{the } \frac{N}{2}\text{-pt. DFT of } \{h(n)\}_{n=0}^{N/2-1}}$$

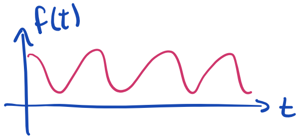
$$X[k] = G[k] + W_N^k H[k] \quad (\text{for } k = 0, 1, \ldots, \tfrac{N}{2}-1)$$

	Naïve	FFT
1D	N^2	$\mathcal{O}(N\log_2 N)$
2D	N^4	$\mathcal{O}(N^2\log_2 N)$

- In MATLAB:
  - 1D signals (functions of time) `fft(x)`
  - 2D signals (images) `fft2(x)`



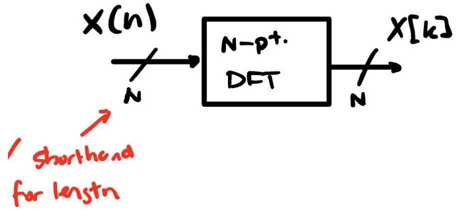
$F(0) = 0$

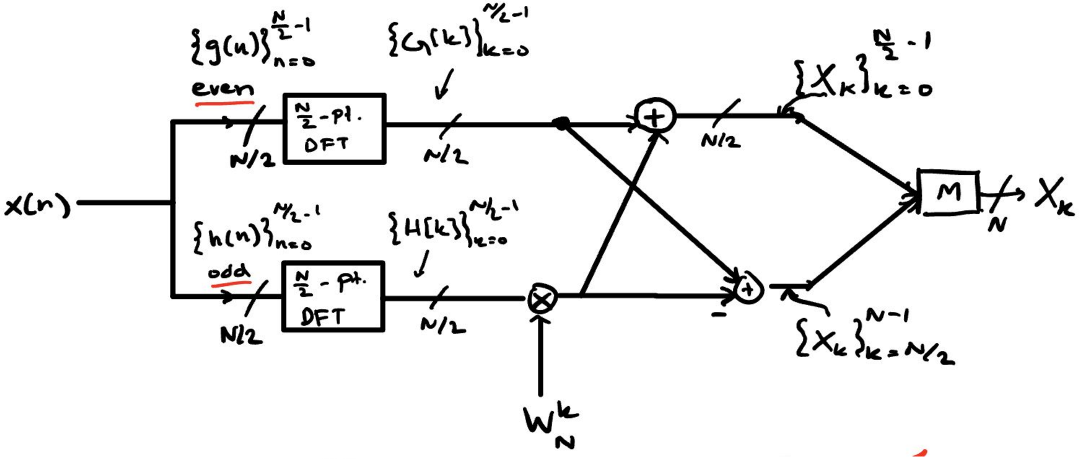


$F(0) > 0$

- F(0) represents the DC component (time-average value of the signal)

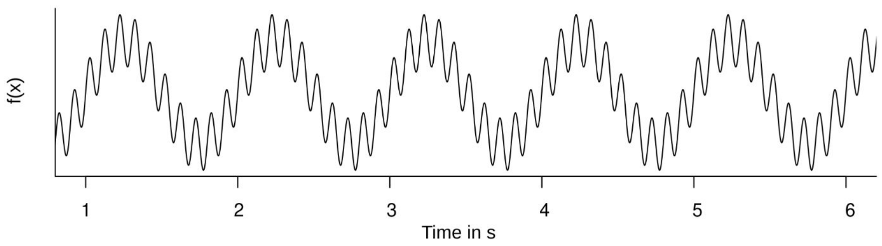
FFT Diagram (Cooley-Tukey) + Example



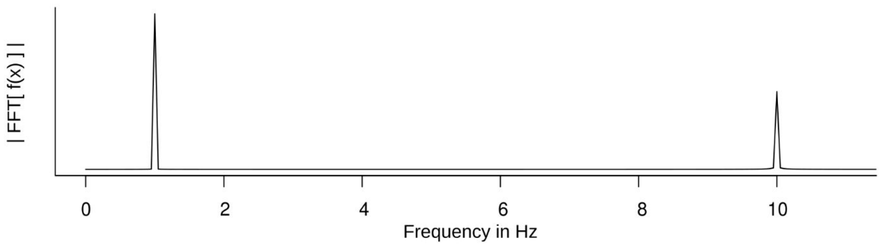


where $W_N^k = e^{-i\frac{2\pi}{N}kn}$

signal f(x) = 2 sin x + sin 10x

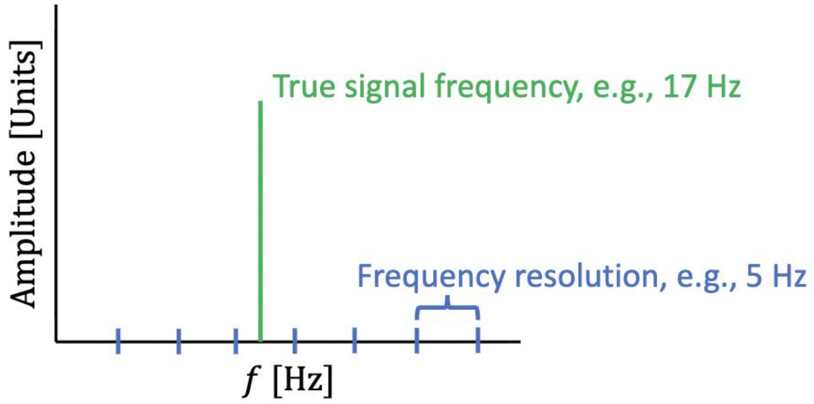


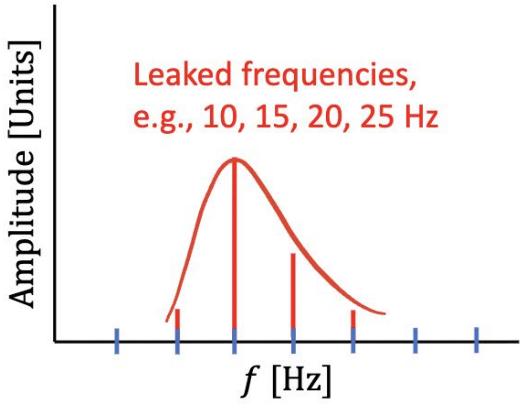
Signal that was fourier-transformed via FFT (frequency spectrum)



Frequency Leakage

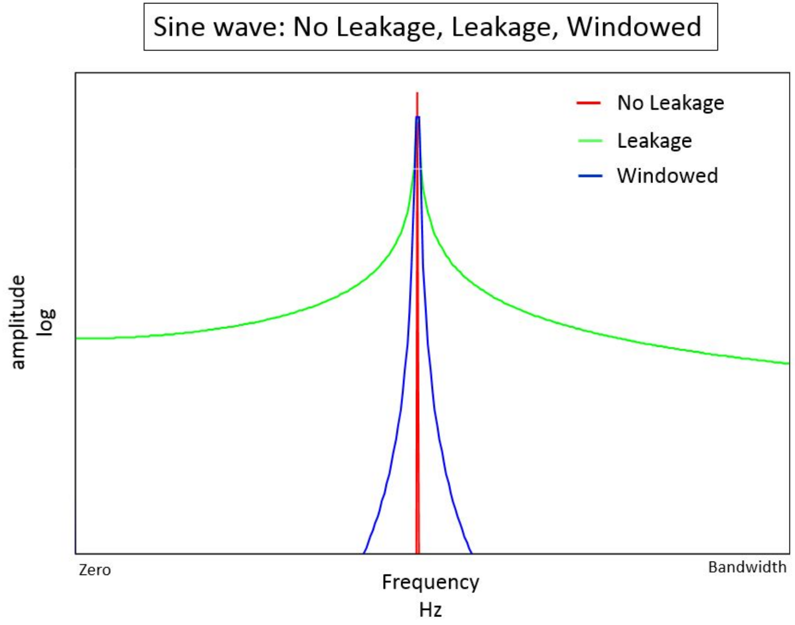
- Signal frequencies don’t always align perfectly with the resolved DFT frequencies, which can only be exact multiples of Δf (i.e. Δf, $2\Delta f$, $3\Delta f$ …. $N/2\ \Delta f$)
- The signal will ‘leak’, or appear in adjacent frequency bins if it lies between resolved frequencies.





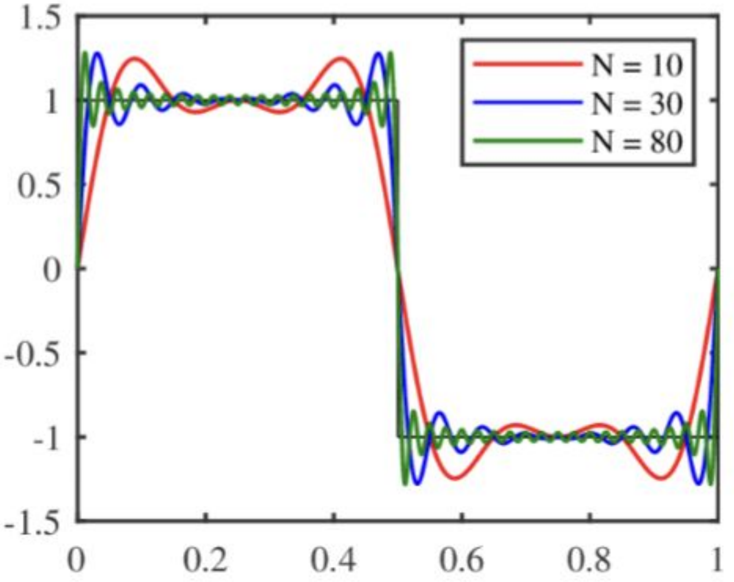
Windowing/ Hann Filter

- Windowing helps minimize this leakage effect.
- Before computing the DFT, multiplying the signal by a function that will smoothly taper the signal to 0 at the boundaries, as opposed to a sharp discontinuity



Gibbs Phenomenon

- Occurs when reconstructing a signal with sharp edges (like a square wave) with a finite number of frequency components
- Since the square wave technically has an infinite number of components, increasing the number of frequency terms creates a "ringing effect" around the edges



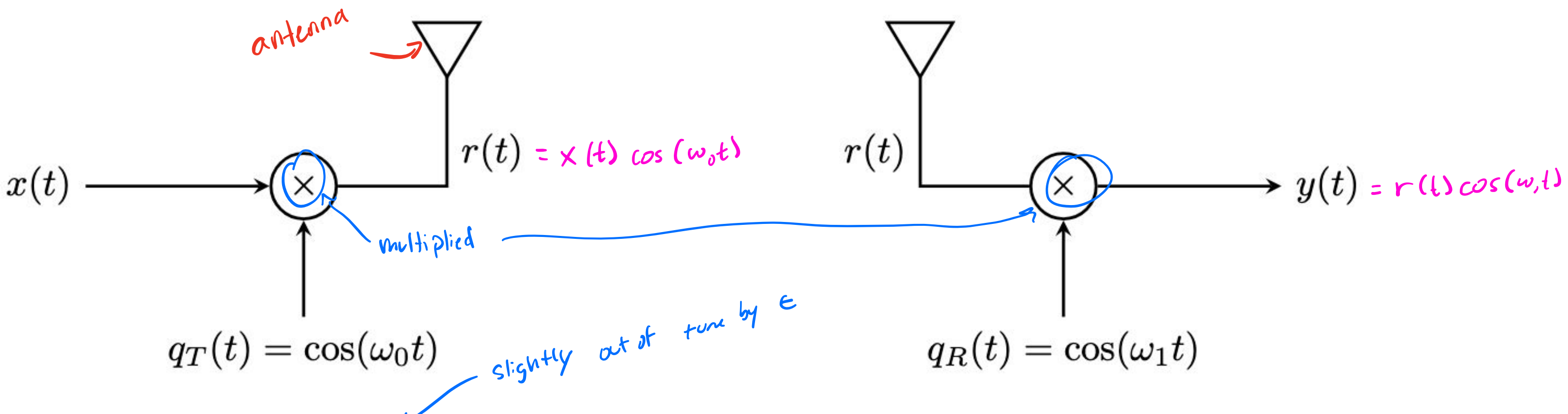
Question 2

Heterodyning and Amplitude Modulation

Amplitude Modulation (AM)

- What is AM?
    - taking a low frequency message and multiplying it by a high frequency carrier wave. [This multiplication in the frequency domain will shift the entire spectrum of our signal from being centered at zero out to +/- ω_0]
- Why do we need AM?
    - to transmit a signal wirelessly, an antenna must be on the same order of magnitude as a signal wavelength. Low frequency signals have massive wavelenths...
- **The scenario in the problem:**
    - transmitter modulates signal using $q_T(t)$, the receiver attempts to demodulate (brings signal back to baseband) but slightly out of tune

Modulation Scheme



There is also a frequency mismatch (ϵ) between the transmitter (LHS) and receiver (RHS) carrier signals q_T and q_R, respectively. In particular, assume that

$$0 < \epsilon \ll A \quad \text{and} \quad A < \omega_0 = \omega_1 + \epsilon$$

bandlimited to A

Fourier Pairs and Properties

From (a) we determined r(t) and y(t). In (b) we need to find R(ω) and Y(ω) which requires us to apply the CTFT. Some helpful properties are

- Modulation Property: $x(t)y(t) \overset{\mathcal{F}}{\longleftrightarrow} \frac{1}{2\pi}(X \circledast Y)(\omega)$

  convolution ≠ multiplication

  $(X * Y)(\omega) = \int_{-\infty}^{\infty} X(\eta) Y(\omega - \eta)\, d\eta$
- Euler's Formula: $\cos(\omega) = \dfrac{e^{j\omega} + e^{-j\omega}}{2}$
- Complex Exponential Fourier Pair: $e^{i\omega_0 t} \overset{\mathcal{F}}{\longleftrightarrow} 2\pi\delta(\omega - \omega_0)$
- Dirac Delta Convolution Property: $f(x) * \delta(x - x_0) = f(x - x_0)$

There are more formulas on bCourses > Files > Statistical Tables and Formulae > Fourier_Z_TransformRefSheet

Example: How to find R(ω)?

① Start with r(t) and take CTFT

We need to apply the modulation property

$$r(t) = x(t)\cos(\omega_0 t) \xleftrightarrow{\mathcal{F}} R(\omega) = \frac{1}{2\pi}\left(X(\omega) * \mathcal{F}\{\cos(\omega_0 t)\}\right)$$

② Determine $\mathcal{F}\{\cos(\omega_0 t)\}$

use Euler's Formula and the complex exponential Fourier pair

$$\cos(\omega_0 t) = \frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}$$

$$\mathcal{F}\{\cos(\omega_0 t)\} = \mathcal{F}\left\{\frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}\right\}$$

$$= \frac{1}{2}\mathcal{F}\left\{e^{j\omega_0 t} + e^{-j\omega_0 t}\right\}$$

$$= \cancel{\frac{1}{2}}\left(\cancel{2}\pi\delta(\omega-\omega_0) + \cancel{2}\pi\delta(\omega+\omega_0)\right)$$

$$= \pi\left(\delta(\omega-\omega_0) + \delta(\omega+\omega_0)\right)$$

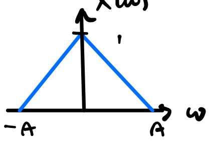
③ Substitute back into convolution + apply δ conv. prop

$$R(\omega) = \frac{1}{2\pi}\left(X(\omega) * \mathcal{F}\{\cos(\omega_0 t)\}\right)$$

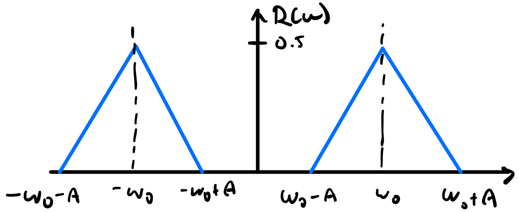
$$= \frac{1}{2\cancel{\pi}}\left(X(\omega) * \cancel{\pi}\left(\delta(\omega-\omega_0) + \delta(\omega+\omega_0)\right)\right)$$

$$\boxed{R(\omega) = \frac{1}{2}\left(X(\omega-\omega_0) + X(\omega+\omega_0)\right)}$$

④ Graph R(ω)!



Remember X(ω) = so we just recenter the triangle and we have 2 copies. We also half the amplitude



Question 3

Frequency Responses of Low Pass Filters

Complex Impedance

- Complex Impedance (usually represented by Z or $\mathbb{Z}$) extends the idea of resistance to AC circuits by including the effects of capacitors and inductors.

$$\mathbb{Z} = R + jX$$

- R – Resistance (real component)
- X – Reactance (imaginary component)
- j – Imaginary Unit

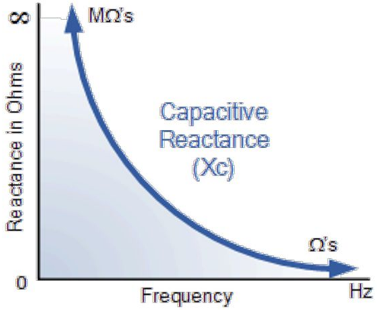
Impedance of Resistors vs. Capacitors vs. Inductors

Resistors: Imepence $\mathbb{Z}$ = R

Capacitors: Impedance decreases as frequency increases

$$\mathbb{Z}_C = \frac{1}{j\omega C} = \frac{1}{j(2\pi f)C}$$

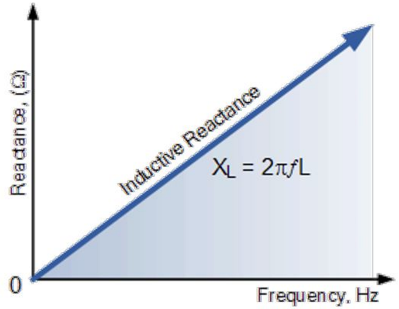
- Low Frequency – Capacitor impedance is high
- High Frequency – Capacitor impedance is low



Inductors: Impedance decreases as frequency increases

$$\mathbb{Z} = j\omega L = j(2\pi f)L$$

- Low Frequency – Inductor impedance is low
- High Frequency – Inductor impedance is high

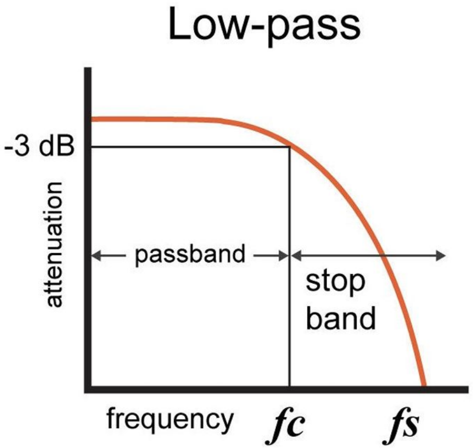


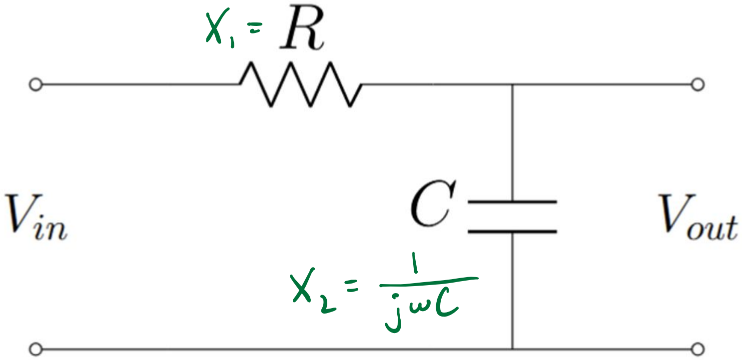
Filter Types - Low Pass

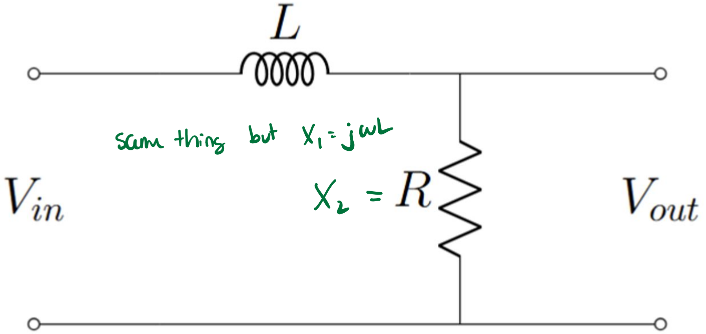
- Filters are circuits that can pass (or amplify) certain frequencies while attenuating other frequencies.
- Low Pass Filters only allows low frequency signals (between 0 Hz to f_c) to pass through. To derive transfer function, use **voltage divider** relation

corner/cutoff frequency

$$\frac{V_{out}}{V_{in}} = \frac{X_2}{X_1 + X_2} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} \cdot \left(\frac{j\omega C}{j\omega C}\right) = \frac{1}{j\omega RC + 1}$$







Filter Types - Higher-Order Filters

Passive First Order Filters are the simplest way to filter a signal.

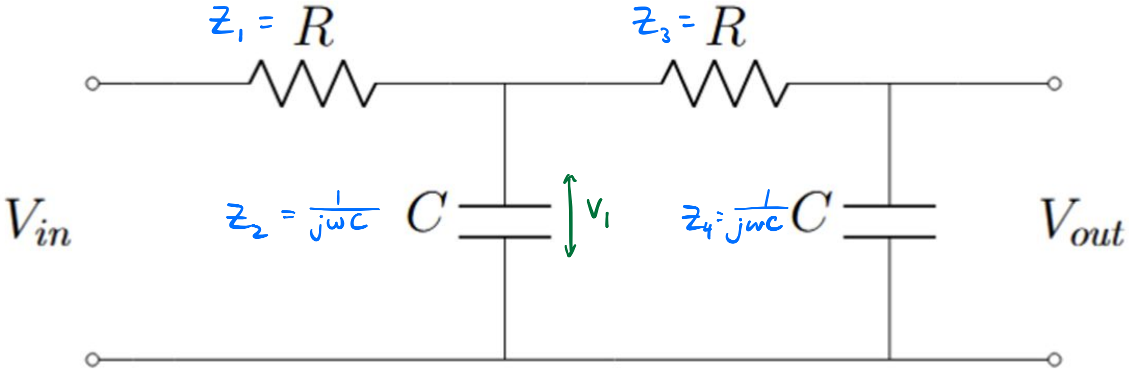
- **Benefits:**
    - Simplicity: requires two components
    - Passive: doesn't require the use of a power supply
- **Limitations**
    - Slow Roll-Off: they aren't "sharp" filters. Attenuates at −20 dB/decade, which does allow nearby frequencies to pass.

Higher-Order Filters have faster or "sharper" roll-off, which translates into a narrower "transition band." They do a better job at separating signals from noise

- 2nd Order: Roll off at -40 dB/decade
- 3rd Order: Roll off at -60 dB/decade
- nth Order: Roll off at n*-20 dB/decade

Filter Types - Higher-Order Filters

What if we were to connect or “stack” two low pass filters to get a second order filter?

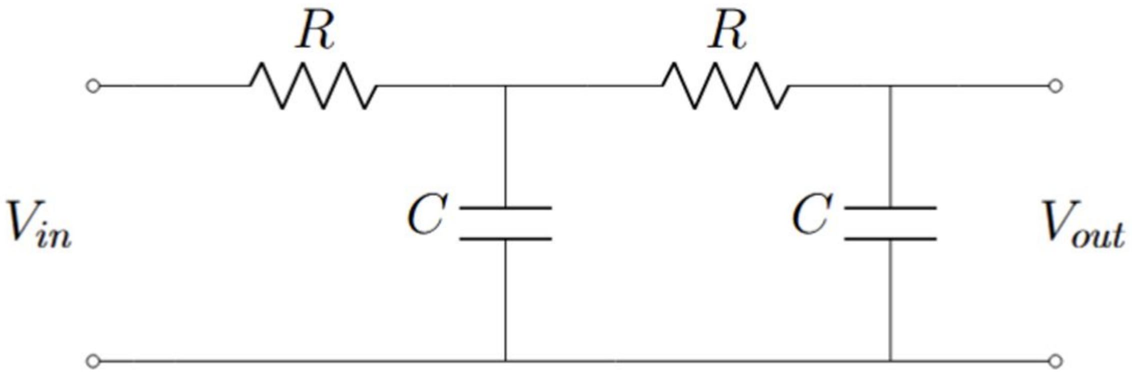


$$Z_5 = \left[(Z_3 + Z_4) \| Z_2 \right] = \left[\frac{1}{R + \frac{1}{j\omega C}} + \frac{1}{\frac{1}{j\omega C}} \right]^{-1}$$

$$\frac{V_1}{V_{in}} = \frac{Z_5}{Z_1 + Z_5} \quad , \quad \frac{V_{out}}{V_1} = \frac{Z_4}{Z_3 + Z_4}$$

$$\Longrightarrow \frac{V_{out}}{V_{in}} = \left(\frac{V_1}{V_{in}}\right)\left(\frac{V_{out}}{V_1}\right) = \left(\frac{Z_5}{Z_1 + Z_5}\right)\left(\frac{Z_4}{Z_3 + Z_4}\right)$$

Filter Types - Higher-Order Filters



if you expand above you will get

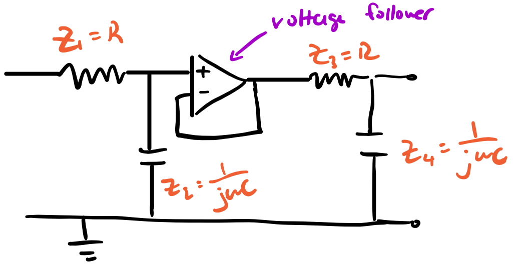
$$\frac{V_{out}}{V_{in}} = \frac{1}{(RC)^2(j\omega)^2 + 3(RC)(j\omega) + 1}$$

$$= \frac{1}{-(RC)^2s^2 + 3RCs + 1}$$

Benefits: faster cut off than 1st order filters

Disadvantages: only good if you want to see a filtered output. There is the "loading" effect otherwise

↳ How to fix?



$$\frac{V_{out}}{V_{in}} = \left(\frac{Z_2}{Z_1 + Z_2}\right)\left(\frac{Z_4}{Z_3 + Z_4}\right)$$

Question 4

Frequency Responses of Other Filters

Phase and Magnitude Responses

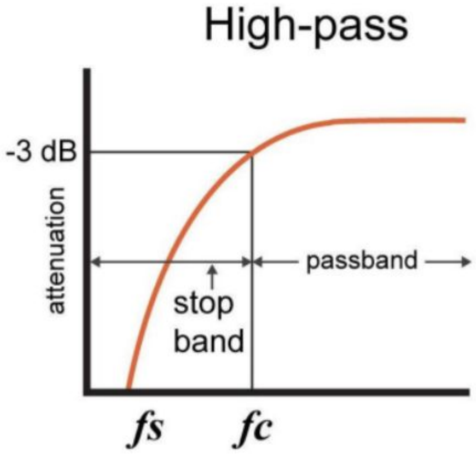
If you have a transfer function as a fraction of two complex numbers $H(j\omega) = \frac{N(j\omega)}{D(j\omega)}$ we can separate the operations

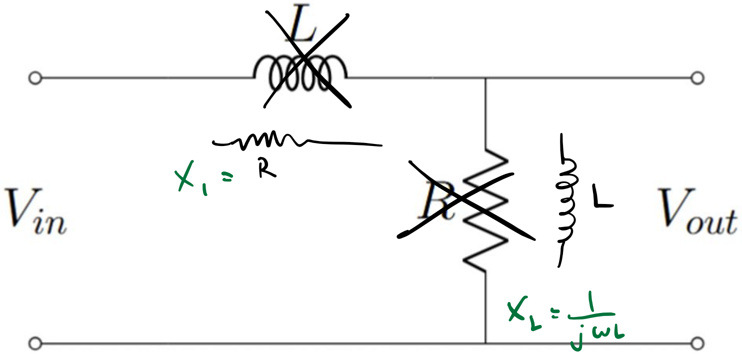
- **Magnitude**: the magnitude of a division is the division of magnitudes $$|H(j\omega)| = \frac{|N(j\omega)|}{|D(j\omega)|}$$
- Recall: for a complex number $a + jb$, the magnitude is $\sqrt{a^2 + b^2}$.
- **Phase:** The phase of a division is the numerator's phase minus the denominator's phase $$\angle H(j\omega) = \angle N(j\omega) - \angle D(j\omega)$$
- Recall: for a complex number $a + jb$, the phase is $\arctan(b/a)$. To account for edge cases, use atan2(y,x).

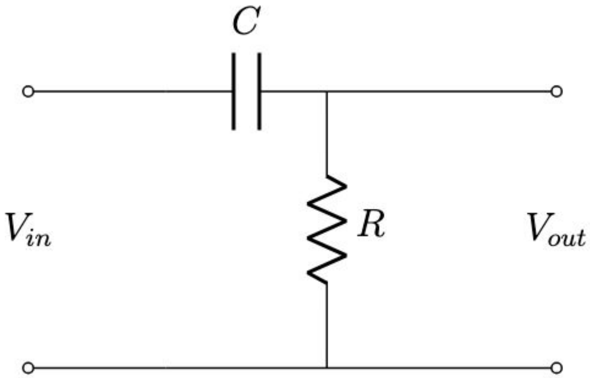
Filter Types - High Pass

- High Pass Filters only allows high frequency signals (between f_c to an infinite frequency) to pass through
- To derive transfer function, use **voltage divider** relation

$$\frac{V_{out}}{V_{in}} = \frac{X_2}{X_1 + X_2} = \frac{j\omega L}{R + j\omega L}$$

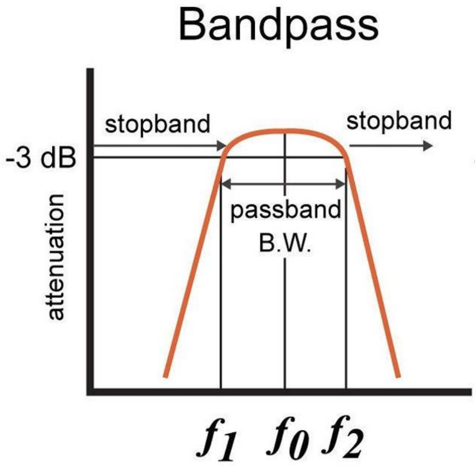


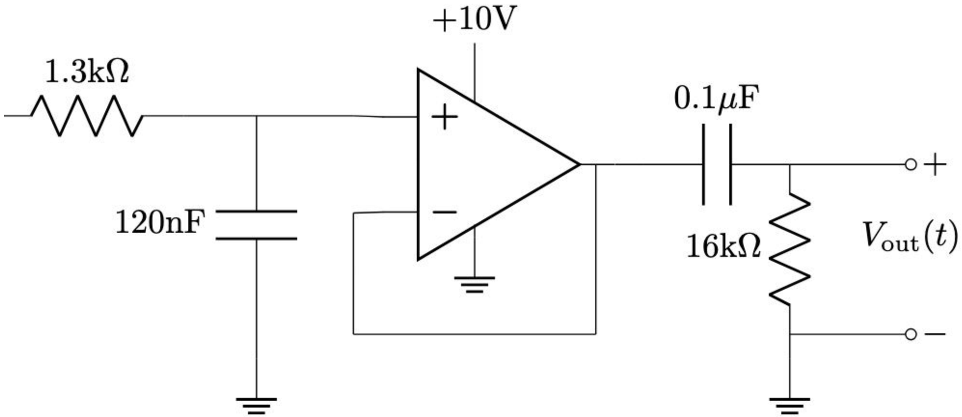




Filter Types - Bandpass

- Band Pass Filters allow only signals within a certain frequency band to pass through



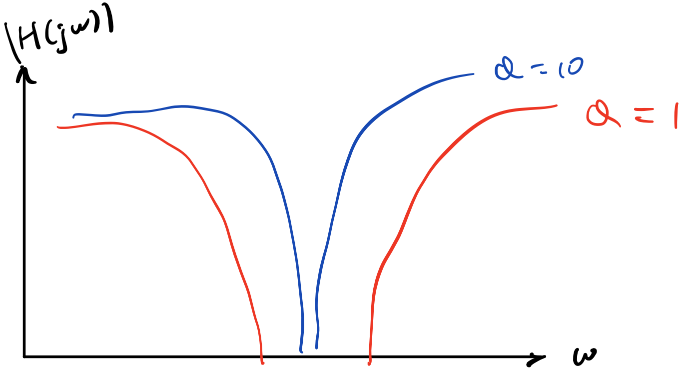


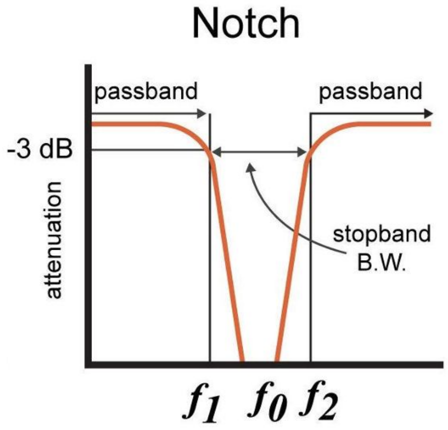
Filter Types - Bandstop/Notch

- Band Stop / Notch Filters allow only signals outside of a certain frequency band to pass through
- The transfer function is below, with Q = quality factor

$$\frac{s^2 + \omega_0^2}{s^2 + Q^{-1}s + \omega_0^2}$$

- Reducing the quality factor would broaden the notch in the frequency spectrum so that a wider range of frequencies is suppressed.





Question 5

Frequency Responses of Low Pass Filters

How to Approach Filtering Questions!

- Translate requirements into numbers and equations!
- Find the required DC gain and convert it into dB

$$K_{dB} = 20\log_{10}(K)$$

- Determine the frequency at which you want to attenuate at.
- Find the total gain you must drop i.e. $\Delta K = K_{DC} - K_{attenuation}$
- Find the total number of decades. For a n-th order low pass, the slope is $n \times -20\ \text{dB}/\text{decade}$. This will tell you how many decades above the cutoff frequency will the attenuation frequency be.
- Use the following to find the cutoff frequency, where N is the decades above the cutoff

$$f_c = \frac{f_{attenuation}}{10^N}$$

Filtering Question Example [ME103 MT, FA25]

A small DC motor, when used as a tachometer, produces a ripple with a peak amplitude of 0.5 V and a frequency that is 100 times higher than the rotational speed being measured. ②

1. [12 pts] Design and draw the circuit diagram for a suitable **active inverting filter**, which attenuates the ripple by 10 dB ③ and amplifies the tachometer voltage by a factor of 2, ① for a measured steady speed of 3000 RPM. Define the values for all resistors in the circuit, assuming only $10\mu F$ capacitors are available.

① DC gain: we want to amplify by a factor of 2 so

$$K_{DC} = 20\log_{10}(2) = 6 dB$$

② Attenuation frequency

$$f_{ripple} = 100 \times f_{rot}$$

$$= 100 \times 3000 \text{ RPM} = \frac{300\,000}{60} \cdot 2\pi$$

$$= 31400 \text{ rad/s} = 4998.59 \text{ Hz}$$

③ Total drop in gain

$$\Delta K = K_{DC} - K_{ripple}$$

from question

$$= 6dB - (-10 dB) = 16 dB$$

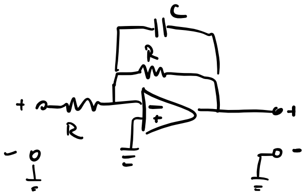
④ Total decades required

$$\frac{\Delta K}{20dB/decade} = \frac{16}{20} = 0.8 \text{ decades} = N$$

⑤ Find cut off

$$f_c = \frac{f_{attenuation}}{10^N} = \frac{4998.59}{10^{0.8}} = \boxed{793 \text{ Hz}}$$

⑥ Circuit Diagram



$$f_c = \frac{1}{2\pi R_2 C_1} \Rightarrow R_2 = \frac{1}{2\pi f_c C_1}$$

$$= \frac{1}{2\pi(793)(10 \cdot 10^{-6})}$$

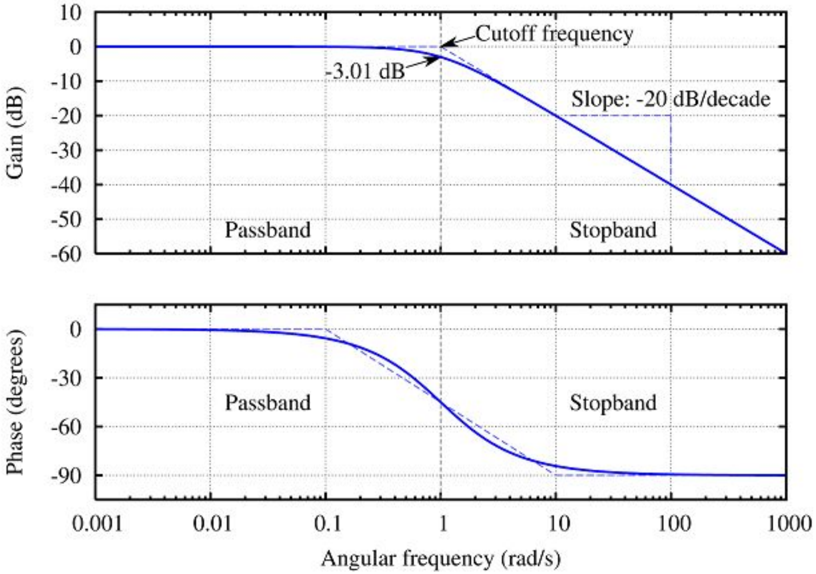
$$\boxed{R_2 = 20\Omega}$$

$$|H|_{DC} = \left| -\frac{R_2}{R_1} \right| = 2 \Rightarrow \boxed{R_1 = \frac{1}{2}R_2 = 10\Omega}$$

Bode Plots

- A **Bode Plot** is a way to visualize how a system (like a circuit or filter) responds to different input frequencies. Consists of two plots:
- **Magnitude Plot (top)**: shows how much the filter amplifies or reduces each frequency. .
  - The gain (y-axis) is reported in dB, which is equal to: $20\log_{10}|H(s)|$
- **Phase plot (bottom):** shows how much the filter delays the output signal at each frequency.
- Given an input signal x(t), TF H(s), and an output signal y(t): $x(t) = \sum_i a_i \cos(\omega_i t)$

$$y(t) = \sum_i a_i |H(j\omega_i)| \cos(\omega_i t + \angle H(j\omega_i))$$

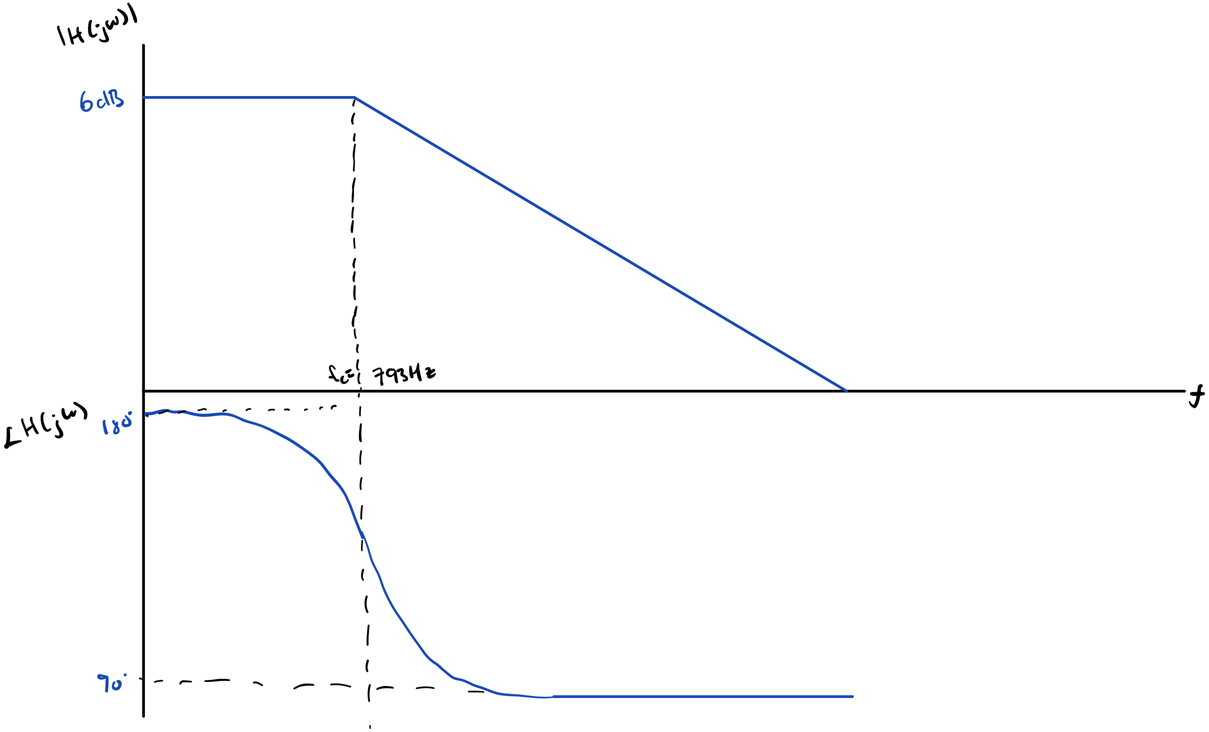


Drawing Bode Plots - Straight Line Approximation

1. **Logarithmic Frequency Axis**
2. **Initial Points**
   a. Calculate the starting magnitude and phase when the frequency is very low (0 Hz)
3. Drawing
   a. **Magnitude Rules (for stable p/z only)**
      i. Every **zero** in the TF (numerator) **adds** 20 db/decade to the slope.
      ii. Every **pole** (zero in the denominator) **subtracts** 20 db/decade
   b. **Phase Rules (for stable p/z, unstable has the rules flipped)**
      i. Every zero increases phase by +90°
      ii. Every pole decreases phase by -90°
      iii. This transition happens over 2 decades (from 0.1ω to 10ω)
4. **Overlap** - if the effects of any pole/zero coincide with another, <u>sum them up.</u>

Filtering Question Example [ME103 MT, FA25]

2. [7 pts] Sketch the Bode magnitude and phase diagrams for this filter, indicating all frequencies, magnitudes and angles of interest.

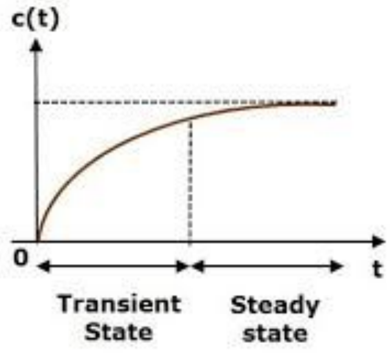


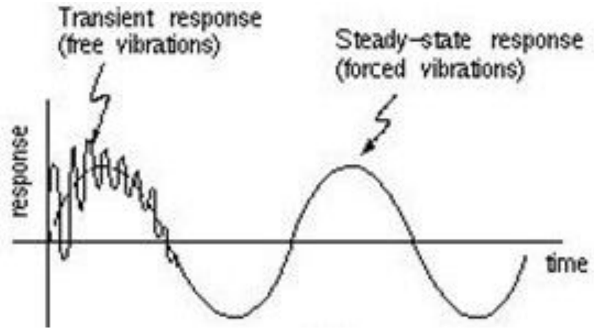
Question 6

Settling Error and Time of ADCs

System Responses: Transient vs. Steady State

- **Transient response**: short term response to a step input **before** the system settles out (i.e. jumps and small oscillations)
- **Steady state**: long term response of the system after the initial transient response. Can vary by time but the characteristics remain unchanged (i.e. amplitude, period, etc.)



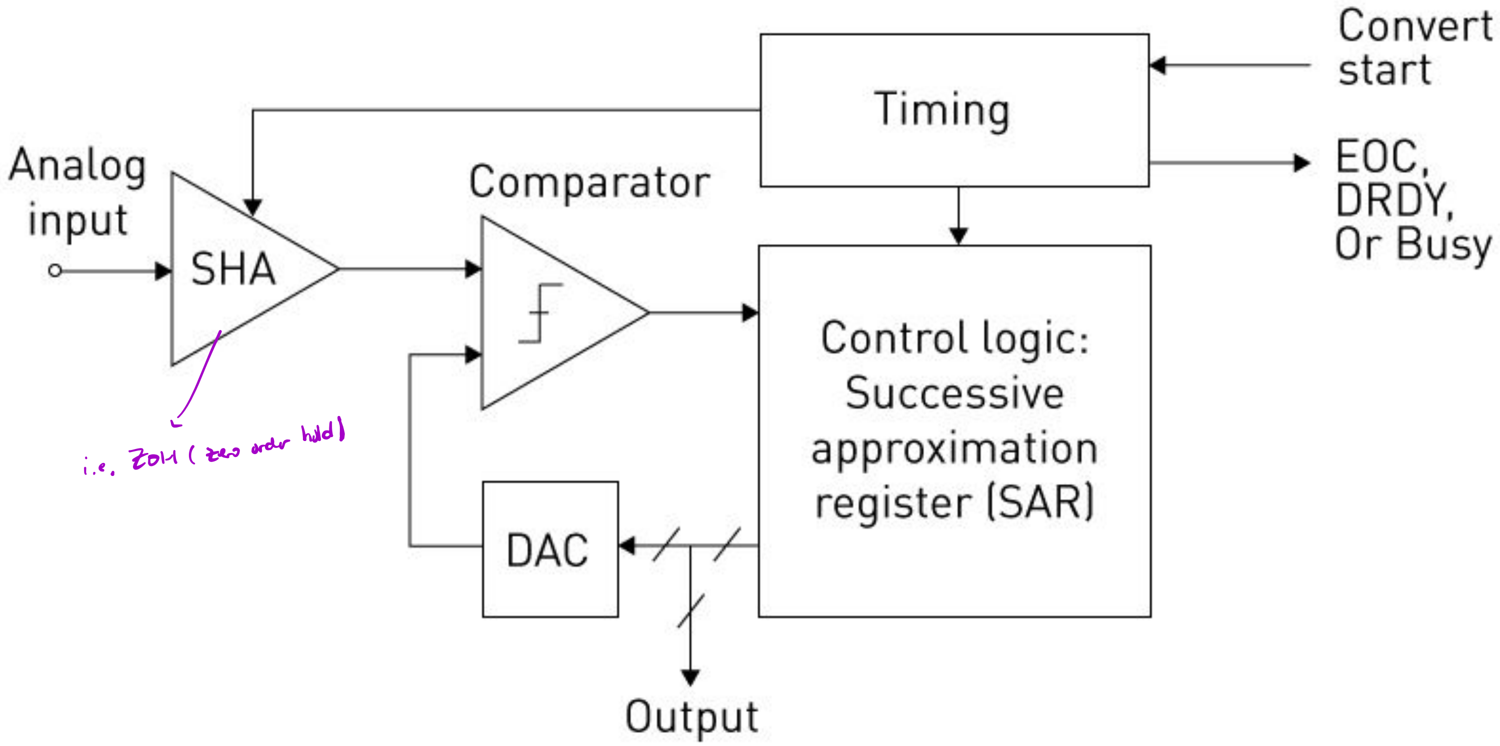


SAR ADC

SAR: Successive Approximation Register. Type of ADC that will figure out the digital value of an analog voltage using a binary search algorithm. How the hardware works:

- **Sample:** samples the continuous analog input voltage and holds it steady on a capacitor using a ZOH or FOH.
- **Guess**: The internal register makes its first guess by setting the Most Significant Bit to 1 (which represents exactly half of the ADC's maximum voltage range).
- **Compare**: An internal DAC turns that guess into a voltage. A comparator checks the real input against the guessed voltage.
- **Refine**: If the real input is higher, the MSB stays a 1. If lower, the MSB flips to a 0. The register then moves to the next bit and repeats the process.

SAR ADC



i.e. ZOH (zero order hold)

Unbuffered Settling Time and ADC Timing

For parts (a) and (b), the following will be helpful. When the 1V step is applied, the ADC input capacitor doesn't charge instantly due to the source resistance R_s.

The voltage at the ADC follows the standard capacitor charging equation

$$v_{\mathrm{SAR}}(t) = V_{\mathrm{step}}\left(1 - \exp\left(-\frac{t}{R_s C_{\mathrm{in}}}\right)\right)$$

To find the time to reach a 1% error, set up an equation for the percentage error *e* (*hint*: $(v_{in}-v_{SAR})/v_{in}$*100%. Set *e* = 0.01 and use natural logs *ln* to isolate and solve for *t*.

SAR ADC Sampling Frequency

For parts (c), recall an SAR ADC determines the voltage one bit at a time. If the ADC has *N* bits, it takes *N* clock cycles to complete one sample.

- Calculate the time for one sample $t_{\text{sampling}} = \frac{N}{f_{\text{clock}}}$
- Invert this to find the maximum sampling frequency

To quote a conservative time, recall the ADC clock runs independently of our input signal. What would happen if a new sampling period starts just before our RC circuit finishes settling to that 1% error margin??

- That first sample will be bad! To guarantee a clean reading, take your RC settling time and add 1 or 2 full ADC sampling periods to it.

Buffering the Input & Slew Rate Limits

- **Why an op-amp buffer?**
  - The original 10 kΩ resistance was causing a massive delay. If we insert a voltage follower, we isolate the source from the ADC.
  - To solve (e), look at the equivalent circuit. The capacitor is no longer driven by the 10 kΩ source resistor; its being driven by the op-amps output resistance
    - calculate the new time constant $\tau = R_{\text{out}} \cdot C_{in}$
    - Use this new time constant to find new 1% settling time
- New RC time constant fast, but op-amps have a physical speed limit called the **slew rate** (V/µs). For (f), calculate the time it takes the op-amp to swing its output by 1V based only on its state slew rate. Compare the slew rate time, new RC settling time, and ADC sampling time → slowest is the bottleneck! Check for LTI 1357 and LM741

General Coding Tips!

Helpful MATLAB tools!

- Here are some helpful MATLAB functions used in this homework!
- For Q1 you may find:
  - `tic` and `toc`: in conjunction used to measure elapsed time. `tic` starts recording, `toc` stops recording.
  - `fft(X)`: computes the DFT of X using a FFT alogrithm.
- For Q3 you may find:
  - `tf(...)`: define a transfer function in the Laplace Domain i.e. *s*-domain
  - `bode(tf)`: plots the Bode magnitude and phase plot given a defined transfer function
  - `hold on`: allows you to plot multiple graphs onto one