

ME 103 Discussion 5

Week of 2/16

Announcements

Tuesday

- Lab 3 Prelab due 7:59 AM ~~Monday~~ next week
- Next week will begin Lab 3 Experiment

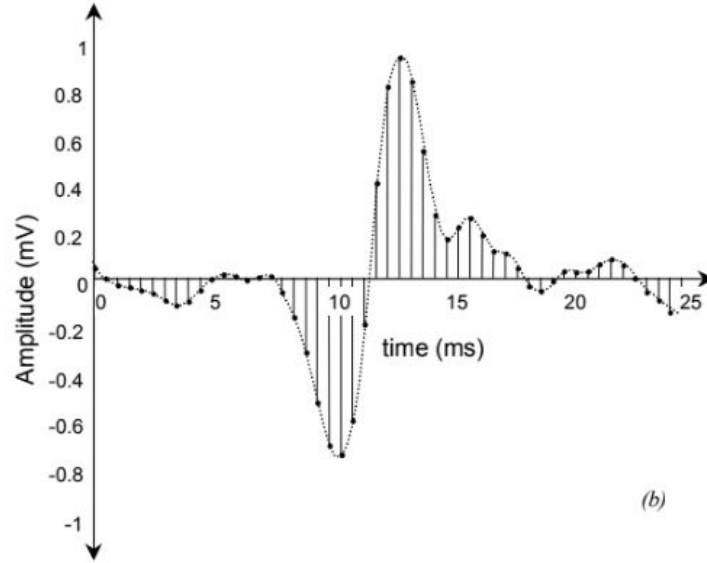
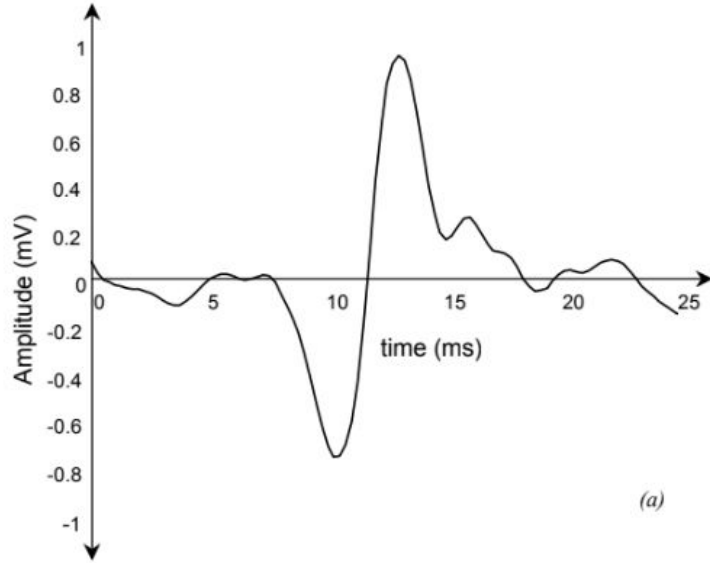
No lab 5 & 4

Vibrating Beam is last in-person lab

Digital Sampling

- When you use the DAQ, you are measuring an analog voltage signal.
- The amplitude of this signal typically varies continuously throughout its range.
- The analog-to-digital conversion (ADC) process generates a sequence of numbers, each number representing the amplitude of the analog signal at a specific point in time.
- This is how we get a representation of this signal on our computer.

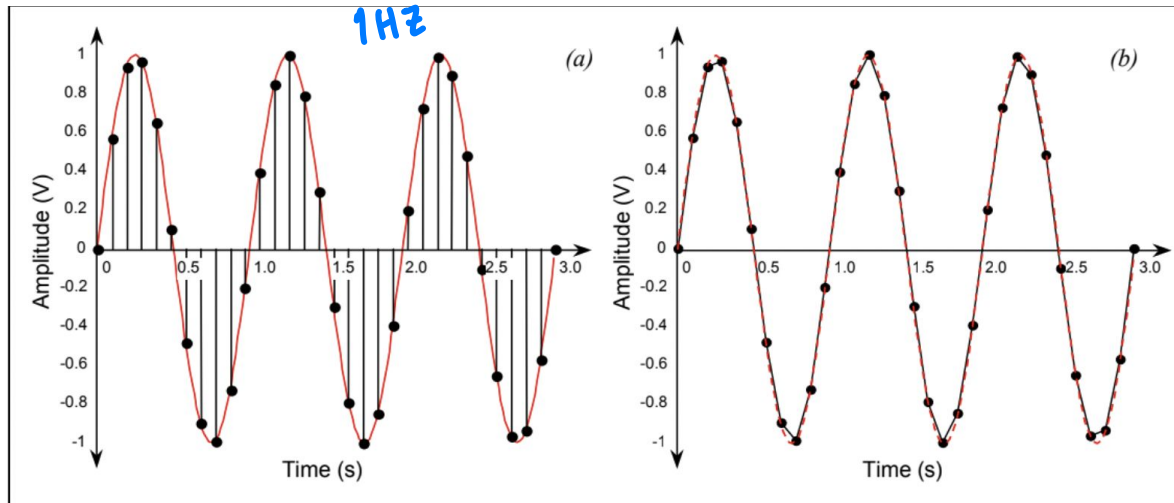
Digital Sampling



- This signal is sampled every 0.5 ms. Another way to describe this is as 2000 samples/second, or 2KHz. This is where we get the term *frequency* from.

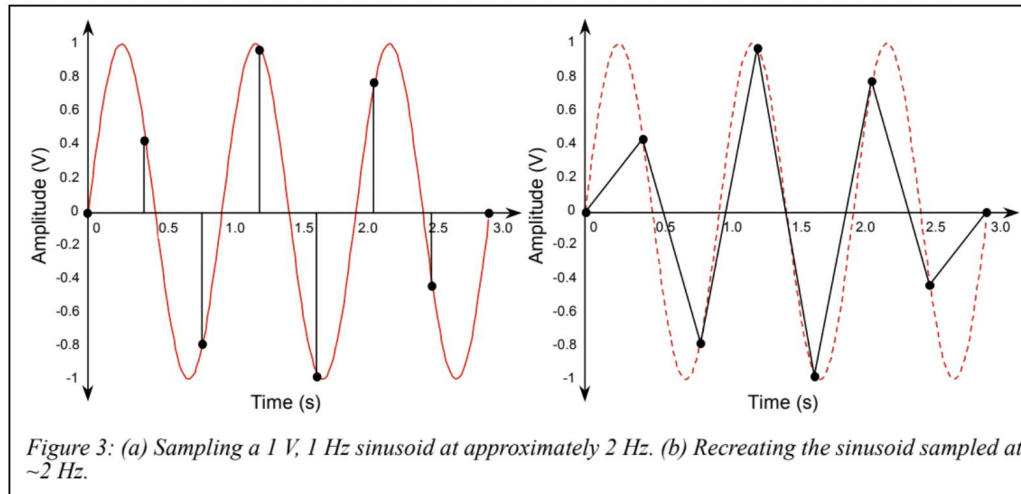
How High Should Sampling Frequency Be?

- We need to sample fast enough so that when we connect the lines between dots it actually looks like the signal.
- Here is a 1V , 1 Hz wave that is sampled at 10 Hz. Clearly there are enough data points to fully recreate the signal.



How High Should Sampling Frequency Be?

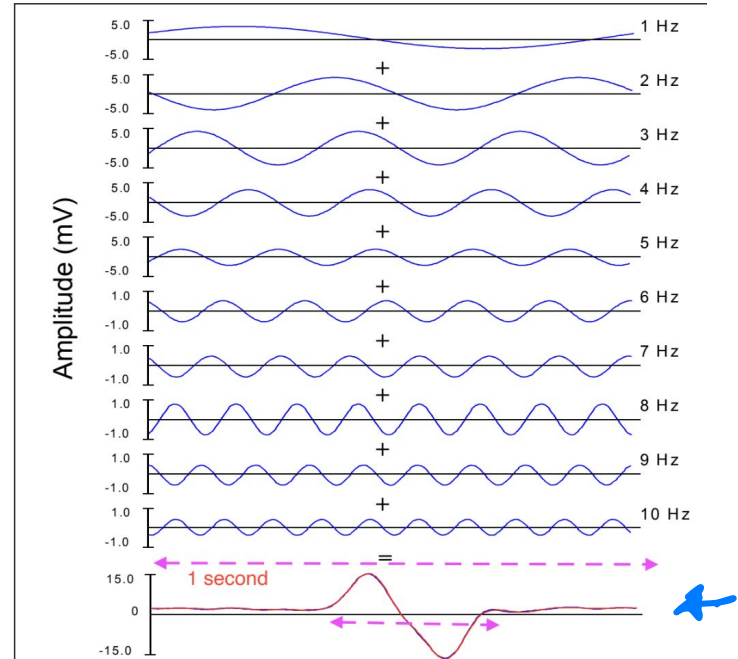
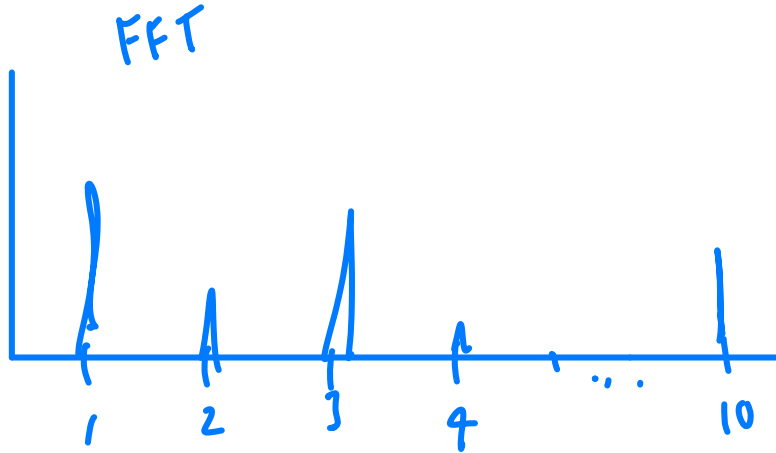
- A sinusoid can only be correctly recreated if it is sampled at no less than twice its frequency. This rule is known as the
- Nyquist Theorem. Violating the Nyquist Theorem (i.e. sampling too slow) leads to an incorrect reconstruction of the signal, typically referred to as *aliasing*, as shown below.



$$f_s > 2f_{max}$$

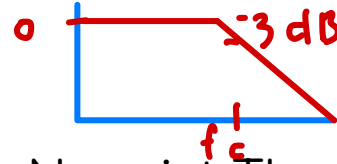
How High Should Sampling Frequency Be?

- For signals with *multiple frequencies*, the Nyquist value must be greater than $2 \times$ the maximum frequency present in the signal. In the photo here, this signal must be sampled at >20 Hz to fully recreate the signal.

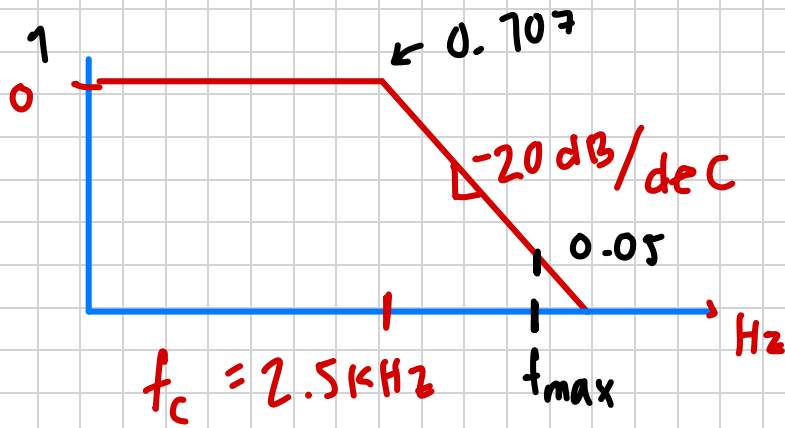


How High Should Sampling Frequency Be?

- -3dB Frequency = cutoff frequency
- Highest frequency: at a gain of 0.05.
- Find the frequency at that gain, and use Nyquist Theorem to determine the sampling frequency
- Conceptual: A problem with a first order low pass filter is that Sampling frequencies in the 100,000 range are resource demanding and inefficient. *How can we fix this?*



- (c) [5 points] Next, suppose that the accelerometer may be sensing some additional harmonics above 2 kHz. To avoid the possibility of aliasing, a first-order low-pass filter is connected between the accelerometer and the A2D board. The -3 dB frequency of the filter is set to 2.5 kHz. The filter can be assumed to cut off signals completely if they occur at frequencies where the gain of the filter is 0.05 or less. What should the sampling rate now become to avoid aliasing?



$$\text{dec} = 10^1 \text{ Hz}$$

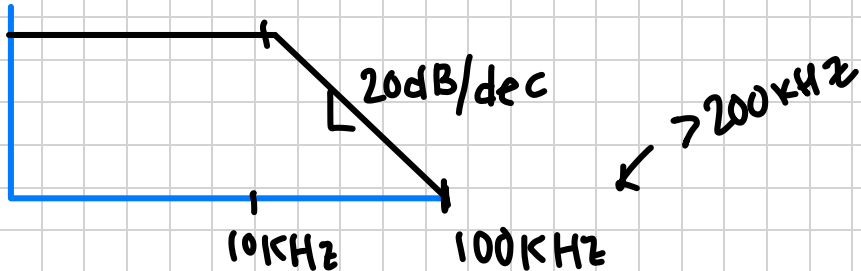
$$\frac{V_{in}}{V_{out}} = 1$$

$$\frac{V_{in}}{V_{out}} = 0.05$$

"-3dB point"

2 KHz

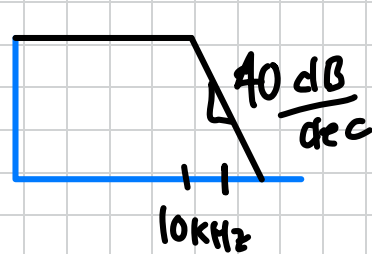
10 KHz



$$\frac{V_{in}}{V_{out}} = H(s) = \frac{1}{\sqrt{1 + \left(\frac{f}{f_c}\right)^{2n}}}$$

$n = \text{order of filter}$

$$n = 1$$



2nd order LP:



Some Impt. Equations:

Low-Pass Filter: $L(f) = \frac{1}{\sqrt{1 + \left(\frac{f}{f_c}\right)^{2n}}}$, where f_c is the 3dB cutoff frequency.

Low Pass Filter Transfer Function (n = order of filter)

$$f_s = \frac{1}{\Delta T}$$

Sampling Frequency
and Period
Relationship

$$f_s \geq 2f_{\max}$$

Nyquist Frequency

$$\Delta f = \frac{f_s}{N}$$

Frequency
Resolution (N= # of
samples)

What Happens when Aliasing Occurs?

6 Manifestation of aliasing [8 points]

$$\begin{array}{l} f_{\max} \quad f_s \\ 100 \text{ Hz} \quad 200 \text{ Hz} \\ \underbrace{\hspace{10em}} \end{array}$$

A 150 Hz cosine wave is sampled at a rate of 200 Hz.

- If you are sampling below the Nyquist Frequency, your apparent signal will occur at $f_s - f_{\max}$. This is why filtering is so important—your frequency readout gets unreadable if this happens.

